

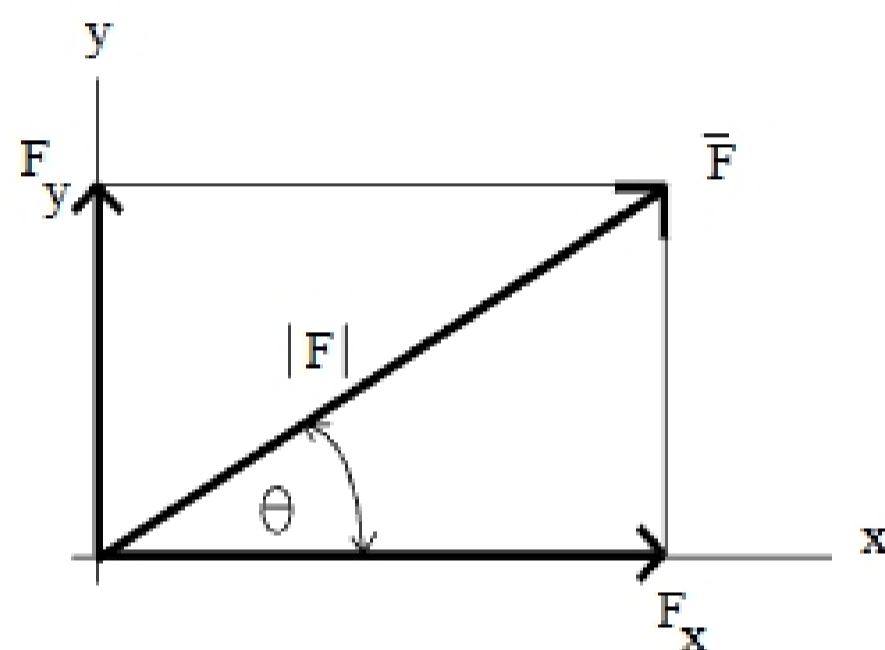
Complex Numbers

The representation of vectors using complex numbers, the conversion between rectangular and polar complex numbers, and calculations using complex numbers will be presented in four ways:

1. by hand
2. using the TI-85/86 calculator
3. using the HP-48G/GX
4. using the TI-89 calculator

Complex Numbers – Background and calculations by hand

Complex number can be used to represent two-dimensional vectors. The complex numbers may be stored in either polar or rectangular form.



Rectangular form: $\bar{F} = F_x \mathbf{i} + F_y \mathbf{j}$

where \mathbf{i} is a unit vector along the x-axis and \mathbf{j} is a unit vector along the y-axis.

Polar form: $\bar{F} = |F| \angle \theta$

where $|F|$ is the magnitude of vector \bar{F} and

θ is the angle of vector \bar{F} measured *counterclockwise from the positive x-axis*

Converting between rectangular form and polar form:

Polar to Rectangular:

$$F_x = |F| \cos(\theta)$$

$$F_y = |F| \sin(\theta)$$

Rectangular to Polar:

$$|F| = \sqrt{F_x^2 + F_y^2} \quad \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

Ex: Convert $\bar{F} = 100 \angle 30^\circ$ N to rectangular form.

$$F_x = 100 \cos(30) = 86.6$$

$$F_y = 100 \sin(30) = 50$$

$$\text{so } \bar{F} = 86.6 \mathbf{i} + 50 \mathbf{j} \text{ N}$$

Ex: Convert $\bar{F} = 30 \mathbf{i} + 40 \mathbf{j}$ N to polar form.

$$|F| = \sqrt{30^2 + 40^2} = 50$$

$$\theta = \tan^{-1} \left(\frac{40}{30} \right) = 53.13^\circ$$

$$\text{so } \bar{F} = 50 \angle 53.13^\circ \text{ N}$$

Complex Numbers using the TI-85 or TI-86

Be sure that the calculator is in degree mode (press 2^{nd} – **MODE** to change the mode).

Complex numbers are stored as follows:

(100/30) represents the polar number $100/30$

(30,40) represents the rectangular number $30i + 40j$

There are two ways to convert between complex number forms:

A) Using mode:

If the calculator is in <u>rectangular mode</u> then the following conversion can be made	
(100/30) <i>Enter</i>	(the value entered in polar form)
(86.6,50)	(the result in rectangular form)

If the calculator is in <u>polar mode</u> then the following conversion can be made	
(30,40) <i>Enter</i>	(the value entered in rectangular form)
(50/53.13)	(the result in polar form)

B) Using the \rightarrow POL and \rightarrow REC functions under the CPLX menu:

(This method works in any mode.)

(100/30)	(the value entered in polar form)
2^{nd} CPLX	
\rightarrow REC <i>Enter</i>	
(86.6,50)	(the result in rectangular form)

(30,40)	(the value entered in rectangular form)
2^{nd} CPLX	
\rightarrow POL <i>Enter</i>	
(50/53.13)	(the result in polar form)

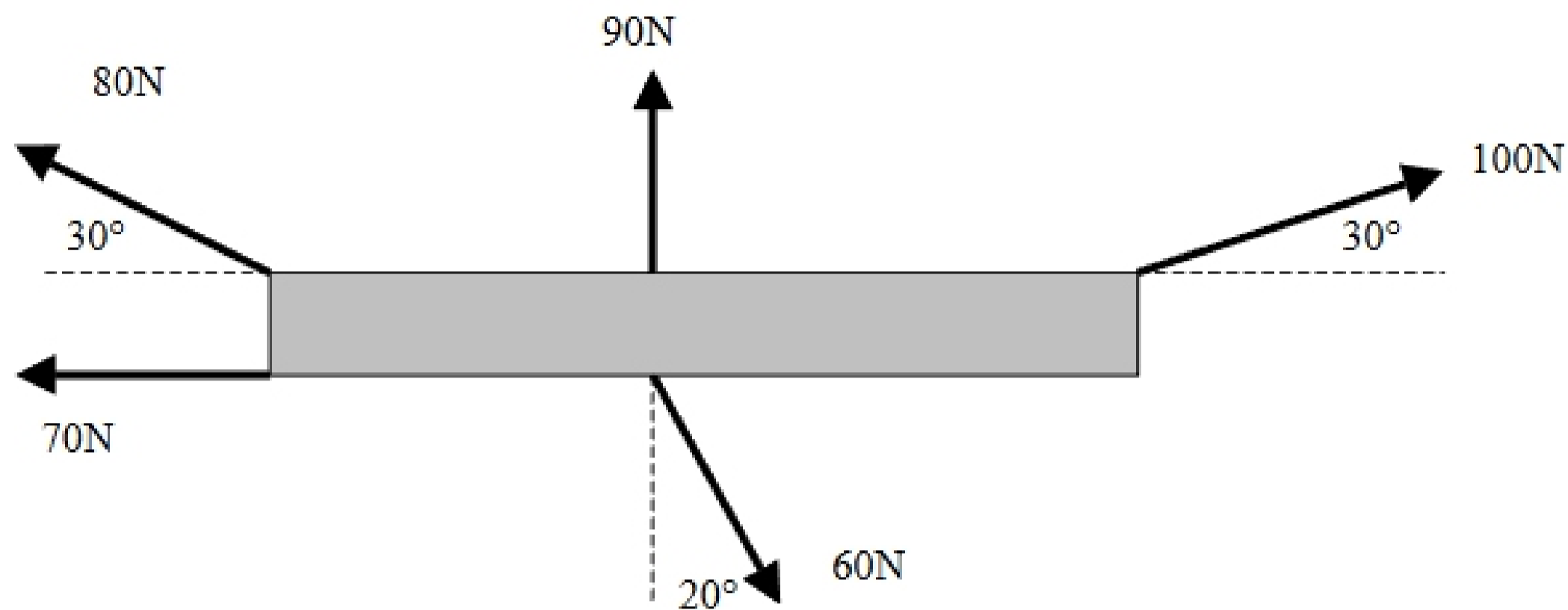
Using variables to store complex numbers with the TI-85/86:

Variables can be used to store complex numbers in either form. For example:

(100/30) STO \rightarrow A	(store this polar number as variable A)
(30,40) STO \rightarrow B	(store this rectangular number as B)
A+B <i>Enter</i>	
(147.3/37.66)	(the result is in whatever form is specified by the mode)

Performing calculations using complex numbers with the TI-85/86:

Calculations can be performed using any combination of real and complex numbers (in any form). **Example:** Find the sum of the forces acting on the beam below:



Recalling that angles in polar numbers must be *measured counterclockwise from the positive x-axis*, the sum of the forces above is easily found as follows:

$(100/_30) + (0,90) + (80/_155) + (-70,0) + (60/_290)$ Enter
 $(122.6/_106.8)$ (the result is in whatever form specified by the mode)

or

$(100/_30) + (90/_90) + (80/_155) + (70/_180) + (60/_-70)$ Enter
 $(122.6/_106.8)$ (the result is in whatever form specified by the mode)