

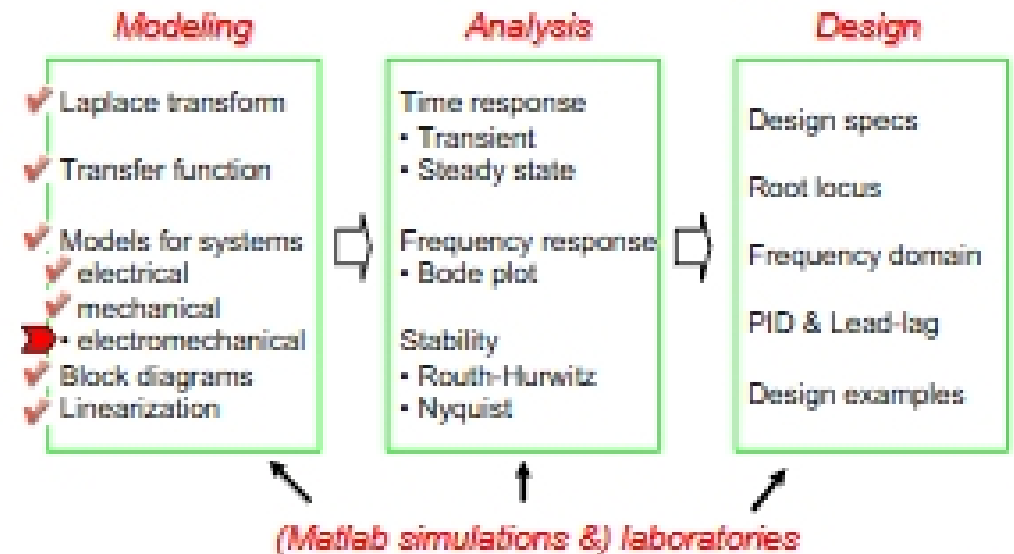
# ME451: Control Systems

## Lecture 8 Modeling of DC motors

Dr. Jongeun Choi  
Department of Mechanical Engineering  
Michigan State University

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## Course roadmap



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## What is DC motor?

*An actuator, converting electrical energy into rotational mechanical energy*



(You will see DC motor during Lab 1 and 4.)

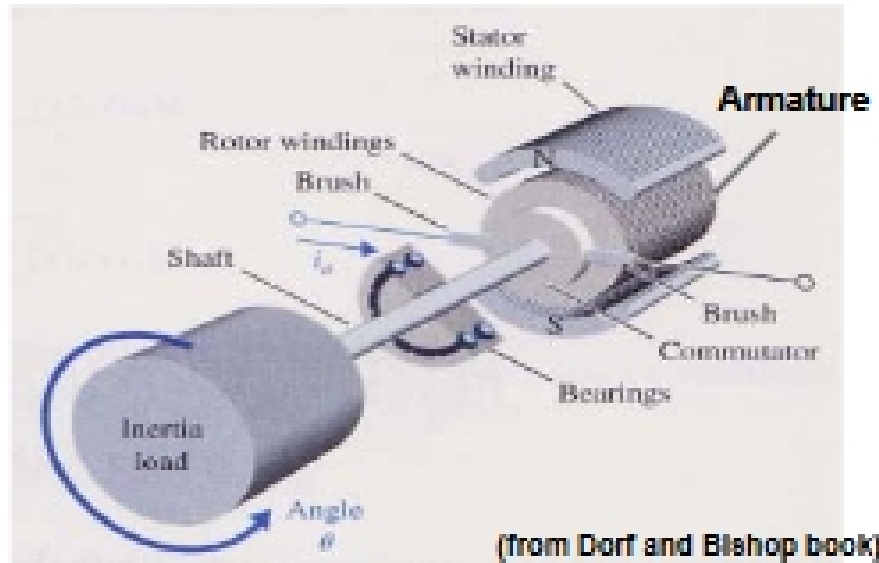
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## Why DC motor?

- Advantages:
  - high torque
  - speed controllability
  - portability, etc.
- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.
- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required

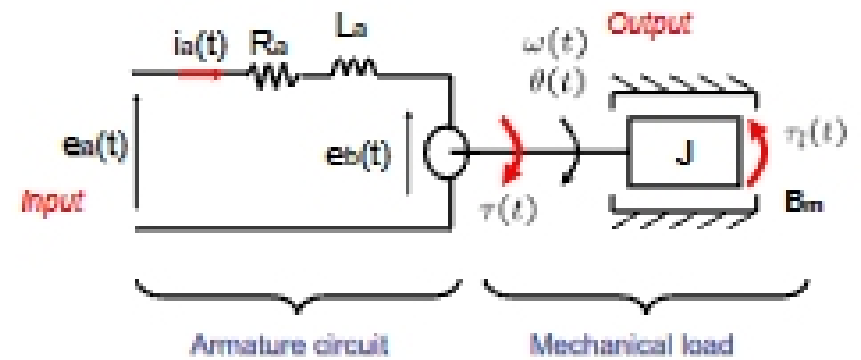
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# How does DC motor work?



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# Model of DC motor



- "a" : armature
- $e_a$  : applied voltage
- $i_a$  : armature current
- "b" : back EMF
- mechanical
- $\theta$  : angular position
- $\omega$  : angular velocity
- $J$  : rotor inertia
- $B$  : viscous friction

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## Modeling of DC motor: time domain

- Armature circuit  $e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$
- Connection between mechanical/electrical parts
  - Motor torque  $\tau(t) = K_\tau i_a(t)$
  - Back EMF  $e_b(t) = K_b \omega(t)$
- Mechanical load  $J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_L(t)$ 

Load torque  
↓
- Angular position  $\omega(t) = \dot{\theta}(t)$

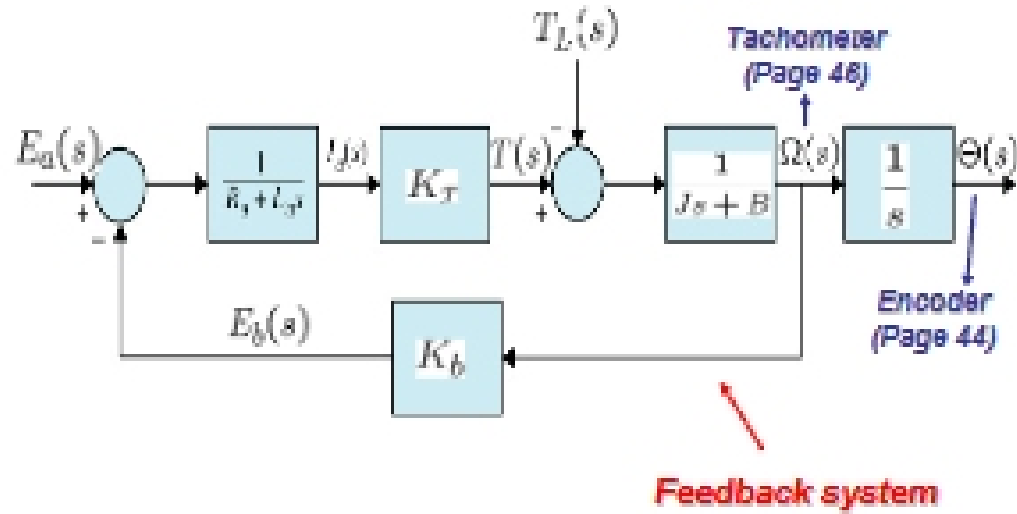
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## Modeling of DC motor: s-domain

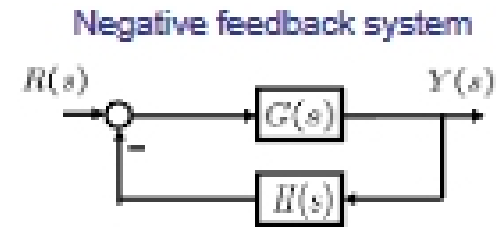
- Armature circuit  $I_a(s) = \frac{1}{R_a + L_a s} (E_a(s) - E_b(s))$
- Connection between mechanical/electrical parts
  - Motor torque  $T(s) = K_\tau I_a(s)$
  - Back EMF  $E_b(s) = K_b \Omega(s)$
- Mechanical load  $\Omega(s) = \frac{1}{J s + B} (T(s) - T_L(s))$
- Angular position  $\Theta(s) = \frac{1}{s} \Omega(s)$

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# DC motor: Block diagram



# Useful formula for feedback



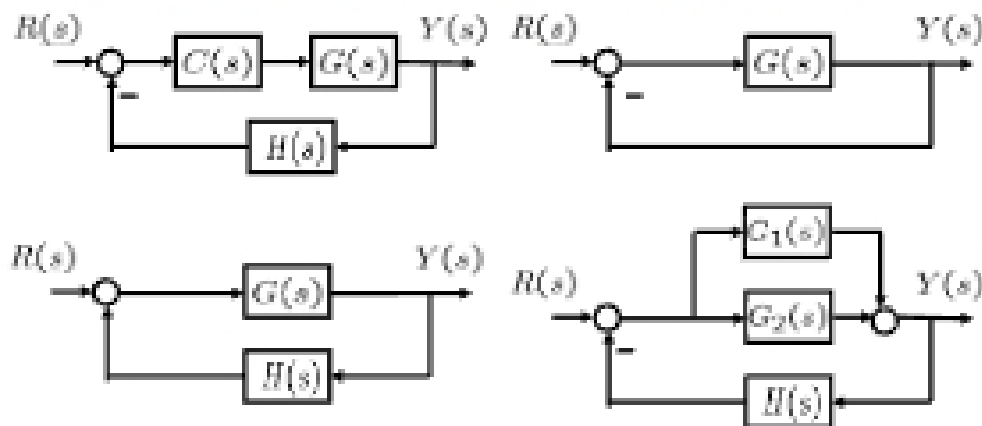
$$Y(s) = G(s)(R(s) - H(s)Y(s)) \implies (1 + G(s)H(s))Y(s) = G(s)R(s)$$

$$\implies \frac{Y(s)}{R(s)} = \frac{F_G}{1 - L_G} = \frac{G(s)}{1 + G(s)H(s)}$$

**Memorize this!**  $\left( \begin{array}{l} G(s) \quad : \text{forward gain} \\ G(s)H(s)(-1) \quad : \text{loop gain} \end{array} \right)$

# Ex: Derivation of transfer functions

Compute transfer functions from  $R(s)$  to  $Y(s)$ .



# DC motor: Transfer functions (TF)

*2<sup>nd</sup> order system*

$$\implies \Omega(s) = G_1(s)E_a(s) + G_2(s)T_L(s)$$

