Matrix Operations

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Linear Combination

A linear combination of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ is defined to be a sum

$$\mathbf{x} = c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k,$$

where $c_1, \ldots, c_k$ are constants.

Vector Algebra

The **norm** or **length** of a fixed vector $\vec{X}$ with components $x_1, \ldots, x_n$ is given by the formula

$$|\vec{X}| = \sqrt{x_1^2 + \cdots + x_n^2}.$$

The **dot product** $\vec{X} \cdot \vec{Y}$ of two fixed vectors $\vec{X}$ and $\vec{Y}$ is defined by

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = x_1y_1 + \cdots + x_ny_n.$$

Angle Between Vectors

If $n = 3$, then $|\vec{X}||\vec{Y}| \cos\theta = \vec{X} \cdot \vec{Y}$ where θ is the **angle between** $\vec{X}$ and $\vec{Y}$. In analogy, two n-vectors are said to be **orthogonal** provided $\vec{X} \cdot \vec{Y} = 0$. It is usual to require that $|\vec{X}| > 0$ and $|\vec{Y}| > 0$ when talking about the angle θ between vectors, in which case we *define* θ to be the acute angle $(0 \le \theta < \pi)$ satisfying

$$\cos\theta = \frac{\vec{X} \cdot \vec{Y}}{|\vec{X}||\vec{Y}|}.$$

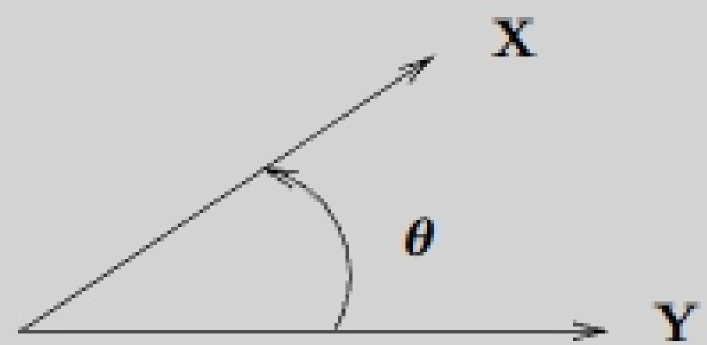

Figure 1. Angle θ between two vectors $\mathbf{X}$, $\mathbf{Y}$.