Chapter 6.2 1-1

Solutions About an "Ordinary Point"

$y(x_0) = y_0 \;;\; y'(x_0) = y_0'$

$a_2(x)\,y'' + a_1(x)\,y' + a_0(x)\,y = 0$

- power series solution would involve $y = \sum_{n=0}^{\infty} c_n (x-a)^n$ → expansion about $x=a$

any point where coefficient $(a_2) = 0$ is called a "singular point". Any point that is not a singular point is called an ordinary points.
→ where $a_2 \neq 0$

- Power Series Solutions about $a=0$
→ $y = \sum_{n=0}^{\infty} c_n x^n$

$y'' + y = 0 \rightarrow y = c_1 \cos(x) + c_2 \sin(x)$

(ex) $y'' + y = 0$

plug in $y = \sum_{n=0}^{\infty} c_n x^n \rightarrow y' = \sum_{n=0}^{\infty} c_n n x^{n-1} = \sum_{n=1}^{\infty} c_n n x^{n-1}$

$y'' = \sum_{n=2}^{\infty} c_n (n)(n-1)\, x^{n-2}$

$$\sum_{n=2}^{\infty} n(n-1)\, c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n} = 0$$

Have / want

$x-2 = m \longrightarrow n = m+2$

$\sum_{m=0}^{\infty} (m+2)(m+2-1)\, c_{m+2}\, x^m$

+ rename $m \rightarrow n \longrightarrow \sum_{n=0}^{\infty} (n+2)(n+1) c_{n+2} x^n + \sum_{n=0}^{\infty} c_n x^n$

$$\Longrightarrow \sum_{n=0}^{\infty} \Big[\overbrace{(n+2)(n+1)\, c_{n+2} + c_n}^{=0}\Big]\, x^n = 0$$

$n = 0, 1, 2 \ldots$

$(n+2)(n+1)\, c_{n+2} + c_n = 0$ [recurrence relation]

- $n=0$ $\;1\cdot 2\, c_2 + c_0 = 0$ $\quad c_2 = -\frac{c_0}{2}$ $\qquad$ - $n=2$ $\;3\cdot 4\, c_4 + c_2 = 0$ $\quad c_4 = \frac{-c_2}{3\cdot 4}$
- $n=1$ $\;2\cdot 3\, c_3 + c_1 = 0$ $\quad c_3 = \frac{-c_1}{2\cdot 3}$

$\longrightarrow \dfrac{+c_0}{2\cdot 3\cdot 4}$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5$$

$$= c_0 + c_1 x - \frac{c_0}{2} x^2 - \frac{c_1}{3!} x^3 + \frac{c_0}{4!} x^4 + \frac{c_1}{5!} x^5$$

$$\therefore y = c_0 \Big[\underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!}}_{\cos(x)} \Big] + c_1 \Big[\underbrace{x - \frac{x^3}{3!} + \frac{x^5}{5!}}_{\sin(x)} \Big]$$