Chapter 2.2

1/22/14

Seperable 1st Order ODE

- $y' = f(x,y)$ → general form of 1st order ODE
- $y' = g(x)h(y)$ * can't be added must be product

$$\int \frac{dy}{h(y)} = \int g(x)\,dx$$

ex) $(1+x)\,dx - y\,dx = 0$

$(1+x)\frac{dy}{dx} - y = 0 \rightarrow \frac{(1+x)\frac{dy}{dx}}{y(1+x)} = \frac{y}{y(1+x)} \rightarrow \frac{\frac{dy}{dx}}{y} = \frac{1}{(1+x)}$

$$\int \frac{dy}{y} = \int \frac{dx}{(1+x)}$$

$\ln|y| = \ln|1+x| + C_1$

$e^{\ln|y|} = e^{\ln|1+x| + C_1}$

$|y| = e^{\ln|1+x|} \cdot e^{C_1} \rightarrow |y| = e^{C_1} \cdot |1+x|$

$1+x>0$ & $y>0$	$1+x>0$ & $y<0$	$1+x<0$ & $y<0$
$y = e^{C_1}(1+x)$	$-y = e^{C_1}(1+x)$	$-y = -e^{C_1}(1+x)$
	or $y = -e^{C_1}(1+x)$	or $y = e^{C_1}(1+x)$

$1+x<0$ & $y>0$

$y = -e^{C_1}(1+x)$

Ans: $y = \pm e^{C_1}(1+x)$

ex) $\frac{dy}{dx} = \frac{-x}{y}$ $\quad y(-1) = -3$

$\int y\,dy = \int -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$ using initial condition $\frac{1}{2}(-3)^2 = -\frac{1}{2}(-1)^2 + C \quad C = \frac{25}{2}$

$y^2 = -x^2 + 25$ implicit solution

$y = \pm\sqrt{-x^2+25}$ explicit solutions

use only $y = -\sqrt{-x^2+25}$ b/c satisfies ODE & initial condition

(ex) $\frac{dy}{dx} = y^2 - 4$

$$\int \frac{dy}{y^2-4} = \int dx$$

$$\int \frac{dy}{(y-2)(y+2)} = \int dx \Rightarrow \int \frac{dy}{(y-2)(x+2)} = x + c$$

$(y-2)(y+2)\left[\frac{1}{(y-2)(y+2)} = \frac{A}{y-2} + \frac{B}{y+2}\right]$

$$= \frac{A(y+2) + B(y-2)}{(y-2)(y+2)}$$

$$= \frac{\overbrace{[A+B]}^{=0} + \overbrace{[2A-2B]}^{=1}}{(y-2)(y+2)}$$

$A+B=0 \rightarrow B=-A$

$2A-2B=1$

$A=\frac{1}{4} \quad B=-\frac{1}{4}$

$$\int \frac{\frac{1}{4}}{y-2} + \frac{-\frac{1}{4}}{y+2}\,dy = \frac{1}{4}\ln|y-2| - \frac{1}{4}\ln|y+2|$$

$$\ln\left|\frac{y-2}{y+2}\right| = 4(x+c) \Rightarrow \frac{y-2}{y+2} = e^{4x}\cdot e^{c} \rightarrow \pm c e^{4x}$$

$$\frac{y-2}{y+2} = Ce^{4x}$$

singular denominator is 0, satisfy original ODE

sol'n

Chapter 2.2 [cont.]

1/24/14

$$\frac{dy}{dx} = y^2 - 4$$

$$\frac{dy}{y^2-4} = dx \Rightarrow \int \frac{dy}{(y-2)(y+2)} = \int dx$$

$$= x + c$$

$y^2-4 \hookrightarrow (y-2)(y+2)$

$$\frac{1}{(y-2)(x+2)} = \frac{A}{y-2} + \frac{B}{y+2} \quad \text{(multiply by } (y-2)(x+2)\text{)}$$

$$= \frac{A(y+2) + B(y-2)}{(y-2)(y+2)} = \frac{y(\overbrace{[A+B]}^{=0}) + (\overbrace{[2A-2B]}^{=1})}{(y-2)(y+2)}$$

$A+B=0 \rightarrow B=-A$

$2A-2B=1$

$A = \frac{1}{4} \quad B = -\frac{1}{4}$

$$\int \frac{\frac{1}{4}}{y-2} + \frac{-\frac{1}{4}}{y+2}\, dy = \frac{1}{4}\ln|y-2| - \frac{1}{4}\ln|y+2|$$

raise to e / ln

$$\ln\left|\frac{y-2}{y+2}\right| = 4(x+c) \rightarrow \left|\frac{y-2}{y+2}\right| = e^{4x} \cdot e^{c}$$

$$y = \frac{2[1+ce^{4x}]}{1-ce^{4x}}$$

$c = 2 \rightarrow y = 2$

Singular solution

- denominator = 0
- satisfy ODE

Check if final solns