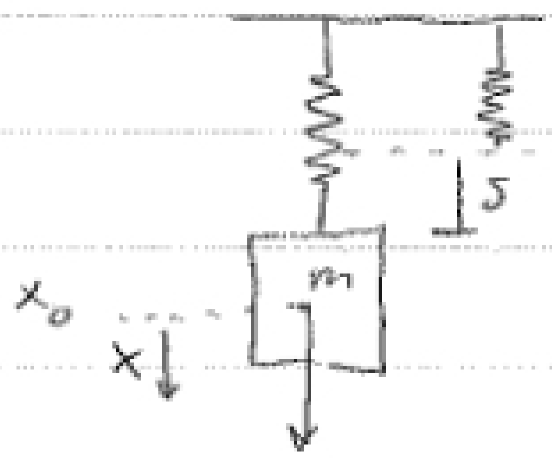
Chapter 5.1: Applications

Spring Mass

· Hookes Law:

① $mg = ks$

② $k = \frac{mg}{s}$



· Newton's 2nd Law: (mass · acceleration) = sum of forces

$$\left(m \cdot \frac{d^2x}{dt}\right) = mg - k(x+s) - c\frac{dx}{dt}$$

$$= mg - kx - \underbrace{ks}_{mg} - c\frac{dx}{dt}$$

$$= -kx - c\frac{dx}{dt}$$

$mx'' + cx' + kx = 0$

└ or $= F_{ext}(t)$ if external forces act

$\boxed{\text{Circuits}}$ $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

(ex) $\boxed{\text{Homogeneous} = \text{no external forces}}$

$mx'' + cx' + kx = 0 \longrightarrow mx'' + kx = 0$

* no drag

$x'' + \frac{k}{m}x = 0$

$y = e^{rt} \rightarrow e^{rt}\left(r^2 + \frac{k}{m}\right) = 0$

$r = \pm\sqrt{\frac{k}{m}}\,i$

$x_1(t) = \cos\left(\sqrt{\frac{k}{m}}\,t\right)$, $x_2(t) = \sin\left(\sqrt{\frac{k}{m}}\,t\right)$

$x(t) = c_1\cos\left(\sqrt{\frac{k}{m}}\,t\right) + c_2\sin\left(\sqrt{\frac{k}{m}}\,t\right)$ $\sqrt{\frac{k}{m}} \rightarrow$ natural frequency

$\sqrt{\frac{k}{m}}\,t$ period for (cos) $= \frac{2\pi}{\sqrt{\frac{k}{m}}}$

$R\left(\cos(A-B)\right) = \left(\underbrace{\cos(A)}_{c_1}\cos(B) + \underbrace{\sin(A)}_{c_2}\sin(B)\right)R$

$c_1 \cos\left(\sqrt{\frac{k}{m}}\,t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}\,t\right)$

$C_1 = R\cos(A)$ $\quad R^2\cos^2 A + R^2\sin^2 A = C_1^2 + C_2^2$

$C_2 = R\sin(A)$ $\quad R^2 = C_1^2 + C_2^2$

amplitude $\longrightarrow$ $R = \sqrt{C_1^2 + C_2^2}$

$$\frac{R\sin(A)}{R\cos(A)} = \frac{C_2}{C_1} \longrightarrow \tan(A) = \frac{C_2}{C_1}$$

$$A = \tan^{-1}\left(\frac{C_2}{C_1}\right)$$

$$x(t) = R\cos\left(A - \sqrt{\tfrac{k}{m}}\,t\right) \longrightarrow R\cos\Big(\underbrace{\sqrt{\tfrac{k}{m}}}_{\omega_0}\,t - \underbrace{A}_{\text{phase } (\phi)}\Big)$$

$$mx'' = F_{spring} + F_{drag} + F_{ext}$$

$$mx'' + \beta x' + kx = F_{ext}(t)$$

$mx'' + kx = 0 \rightarrow$ Free, undamped motion

$$x'' + \frac{k}{m}x = 0$$

$$x = e^{rt}$$

$$\rightarrow e^{rt}\left[r^2 + \frac{k}{m}\right] = 0$$

natural frequency (ω)

$$r = \sqrt{\frac{k}{m}}\, i \qquad x_1 = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$x_2 = \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$x = c_1 \cos\left(\sqrt{\frac{k}{m}}\, t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$\sin\left(\sqrt{\frac{k}{m}}\, t + \phi\right) \quad R\sin\left(\sqrt{\frac{k}{m}}\, t\right)\cos(\phi) + R\cos\left(\sqrt{\frac{k}{m}}\, t\right)\sin(\phi)$$

c_2 c_1

$$c_1 = R\sin(\phi) \qquad c_2 = R\cos(\phi)$$

$$c_1^2 + c_2^2 = R^2 \rightarrow \underbrace{R = \sqrt{c_1^2 + c_2^2}}_{\text{amplitude}}$$

$$\frac{c_1}{c_2} = \tan(\phi) \rightarrow \phi = \tan^{-1}\left(\frac{c_1}{c_2}\right)$$

coeff cos

coeff sin

$\rightarrow$ phase

period: $\frac{2\pi}{\omega}$

$\boxed{\text{Free Damped Motion}} \rightarrow mx'' + \beta x' + kx = 0$

$$y = e^{rt}$$

$$\rightarrow e^{rt}(mr^2 + \beta r + k) = 0$$

$$r = \frac{-\beta \pm \sqrt{\beta^2 - 4mk}}{2m} \rightarrow -\underbrace{\frac{\beta}{2m}}_{\lambda} \pm \sqrt{\underbrace{\frac{\beta^2}{4m^2}}_{\lambda^2} - \underbrace{\frac{k}{m}}_{\omega^2}}$$

$$= -\frac{\lambda}{m} \pm \sqrt{\frac{\lambda}{m^2} - \omega^2}$$