Chapter 3.1 Examples 2/12/14

(2) $\frac{dP}{dt} \propto P \longrightarrow \frac{dP}{dt} = kP \;;\; P(0) = P_0$

$P(3\text{ yrs}) = 10{,}000$

population doubles in 5yrs

$P(5) = 2P_0$

$$\int \frac{dP}{P} = \int k\,dt$$

$\ln|P| = kt + c$

$P = e^{kt+c} = e^{kt} e^{c}$ $\bar{c}$

$P(t) = \bar{c}e^{kt}$

$P(0) = \bar{c} = P_0 \implies P(t) = P_0 e^{kt}$

$P(5) = P_0 e^{5k} = 2P_0$

$e^{5k} = 2$

$5k = \ln|2| \longrightarrow k = \frac{\ln|2|}{5} \implies P(t) = P_0 e^{\left(\frac{\ln|2|}{5}\right)t}$

$P(3) = P_0 e^{\left(\frac{\ln(2)}{5}\right)(3)} = 10{,}000$ * Solve for P_0

$P_0 = 10000 e^{-\frac{3}{5}\ln(2)}$

$e^{\ln(2)^{-\frac{3}{5}}} = 2^{-\frac{3}{5}}$

$= 10{,}000 \left(2^{-\frac{3}{5}}\right)$

(6) $\frac{dA}{dt} \propto A \longrightarrow \frac{dA}{dt} = -kA$

$A(0) = 100mg$

$A(6) = 97\%$ of $100mg = 97mg$

Find $A(24)$

$$\int \frac{dA}{A} = \int -k\,dt$$

$\ln|A| = -kt + c$

$A = e^{-kt+c}$

$= e^{-kt} e^{c}$ $\bar{c}$

$A(t) = \bar{c}e^{-kt}$

$A(0) = \bar{c}(1) = 100mg$

$A(t) = 100e^{-kt}$

$A(6) = 100e^{-6k} = 97$

$e^{-6k} = .97 \longrightarrow -6k = \ln|.97| \longrightarrow k = \frac{\ln|.97|}{-6}$

$A(t) = 100e^{\frac{\ln|.97|}{6}t}$

$A(24) = 100e^{\left(\frac{\ln|.97|}{6}\right)24}$

$= 100^{e^{4\ln|.97|}} = 100(.97)^4$

Half-Life

$A(t_{1/2}) = A_0 e^{-kt_{1/2}} = \frac{A_0}{2}$

$-kt_{1/2} = \underbrace{\ln\left|\tfrac{1}{2}\right|}_{= \ln|1| - \ln|2|}$

$\ln|1| = 0$

$t_{1/2} = \frac{\ln|2|}{k}$

(14) $\frac{dT}{dt} = (T - T_{surr})k$; $T(0) = T_0$

$T_{surr} = 5°F$ $T(1) = 55°$; $T(5) = 30°$

$\int \frac{dT}{T-5} = \int k\,dt$

$\ln|T-5| = kt + c \longrightarrow T-5 = e^{kt+c}$

$= e^{kt}e^{c} = \bar{c}$

$T-5 = \bar{c}e^{kt}$

Chapter 3.1 [cont.]

2/12/14

$T(t) = 5 + \bar{c}e^{kt}$

$T_0 = 5 + \bar{c}(1) = T_0$

$\bar{c} = T_0 - 5 \longrightarrow T(t) = 5 + (T_0 - 5)e^{kt}$

$T(1) = 5 + (T_0 - 5)e^{k} = 55$

$T(5) = 5 + (T_0 - 5)e^{5k} = 30$

$\underline{50 = (T_0 - 5)e^{k}}$ $\qquad 25 = (T_0 - 5)e^{5k}$

$= 2 = e^{-4k}$

$\ln|2| = -4k \longrightarrow k = \frac{-\ln(2)}{4}$

$50 = (T_0 - 5)e^{-\frac{\ln(2)}{4}} \longrightarrow e^{-\frac{1}{4}\ln(2)} \longrightarrow 2^{-\frac{1}{4}} = \frac{1}{2^{\frac{1}{4}}}$

$50 = (T_0 - 5)\frac{1}{2^{\frac{1}{4}}} \longrightarrow T_0 = 5 + 50 \cdot 2^{\frac{1}{4}}$