

Lecture Outline (More Implicit Differentiation)

Friday, February 15

Announcements

1. No Class or OH on Monday, 2/18/08 - holiday
2. Quiz 5 due on Wednesday, 2/20/08.
3. Homework 5 due on Wednesday, 2/20/08.

Recap

Last time we began learning about implicit differentiation.

Recall how we do this:

1. start with a complicated expression involving x 's and y 's; this makes y an implicit function of x .
2. compute the derivative of both sides of this expression (don't forget those $\frac{dy}{dx}$'s which pop up!)
3. solve for $\frac{dy}{dx}$
4. celebrate, because $\frac{dy}{dx}$ gives you the slope of the tangent line to the curve by your original complicated expression

Examples

Let's do some more examples.

1. Find y' where $x^2 + xy + y^3 = 3$. Now find the equation of the tangent line that passes through $(1, 1)$.
2. Find the equation of the tangent line to $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$.
3. Find the derivative of $\ln(x)$. You can memorize the formula you get.
4. Find the derivative of $y = \arctan(x)$. You can memorize the formula you get.

Motivation

Over the last few weeks we've gotten really good at taking derivatives of functions and equations. We've used this skill to be able to find the equation of the tangent line to just about any curve (which tells us the instantaneous rate of change). Derivatives have many more applications than just this.

The application that we are going to start talking about can be described as the "shape of the curve". If we have a function $f(x)$ with some physical meaning (maybe $f(x)$ is a population at time x), we want to be able to compute where $f(x)$ is increasing and decreasing, where the maximum and minimum points are, and other information. After talking about this in a theoretical setting, we will apply this to optimization problems, where we try to maximize or minimize some quantity (revenue, for example.)

Definitions

Let's talk about some basic ways to describe the graph of a function $f(x)$.

A function $y = f(x)$ is **increasing** on an interval if, whenever $a < b$ are numbers in the interval, $f(a) < f(b)$. Less formally, a function $f(x)$ is increasing on an interval if bigger values of x give bigger values of $f(x)$.

A function $y = f(x)$ is **decreasing** on an interval if, whenever $a < b$ are numbers in the interval, $f(a) > f(b)$. Less formally, a function $f(x)$ is decreasing on an interval if bigger values of x give smaller values of $f(x)$.

How does this relate to $f'(x)$? Notice that if $f(x)$ is increasing on an interval, the slope of the tangent line at any point in that interval is positive. We can check this with the limit definition of the derivative. Thus if $f(x)$ is increasing, $f'(x) > 0$.

Similarly, if $f(x)$ is decreasing on an interval, $f'(x) < 0$.