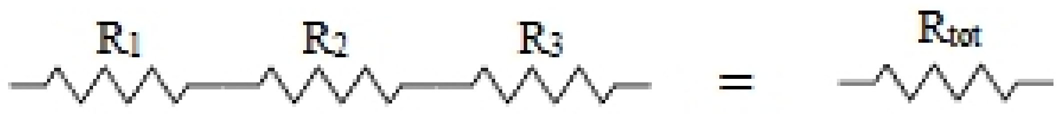
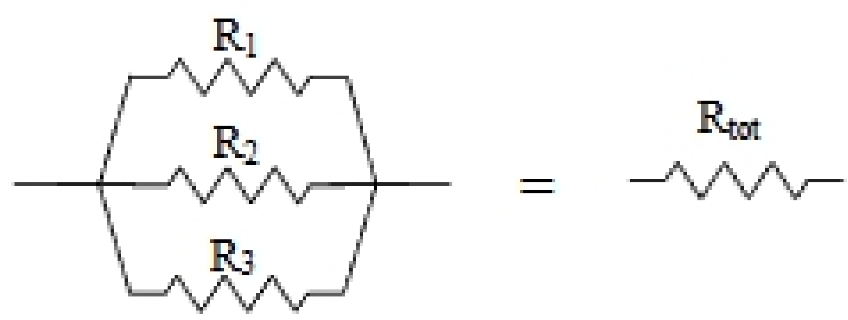
More Circuits

In an electrical circuit, circuit elements such as resistors and batteries can be connected together *in series* or *in parallel*. Resistors in series are connected like links in a chain; resistors in parallel are side-by-side, like so:

in series:



in parallel:



Series: $R_{tot} = R_1 + R_2 + R_3$, $R_{tot} > R_1, R_2, R_3$

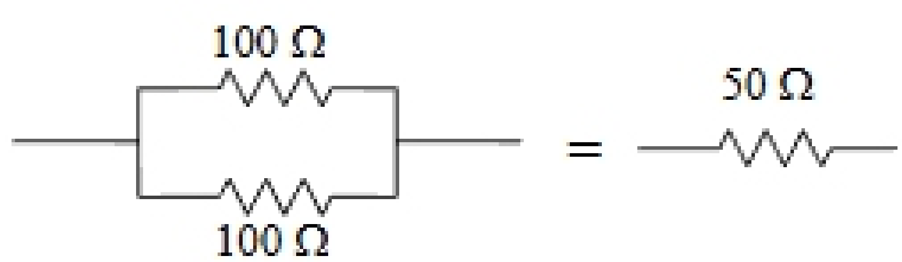
Parallel: $R_{tot} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$, $R_{tot} < R_1, R_2, R_3$

Resistors in series act like a single large resistor.

Resistors in parallel act like a single small resistor.

Examples of parallel resistors:

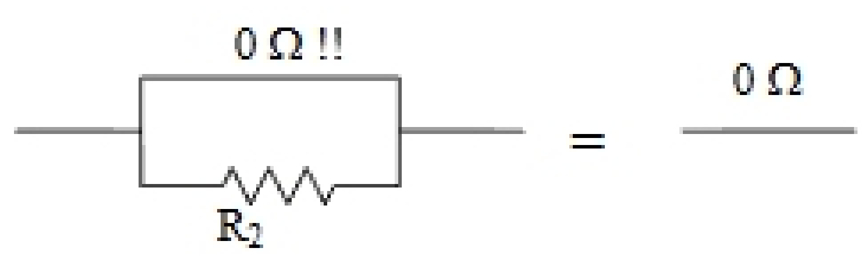
1) Two 100 Ω resistors in parallel:



$$R_{tot} = \frac{1}{\dfrac{1}{100\,\Omega} + \dfrac{1}{100\,\Omega}} = \frac{1}{\left(\dfrac{2}{100}\right)} = \frac{100}{2} = 50\Omega$$

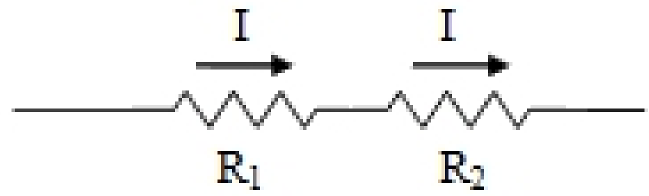
2) 10 Ω in parallel with 0 Ω wire:

$$R_{tot} = \frac{1}{\dfrac{1}{0} + \dfrac{1}{10}} = \frac{1}{\infty} = 0$$

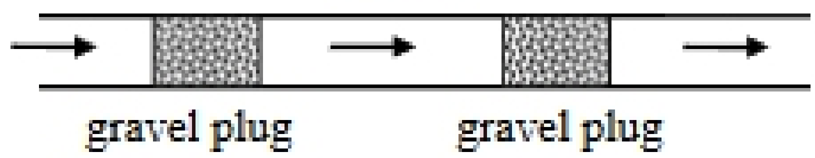


Key points:

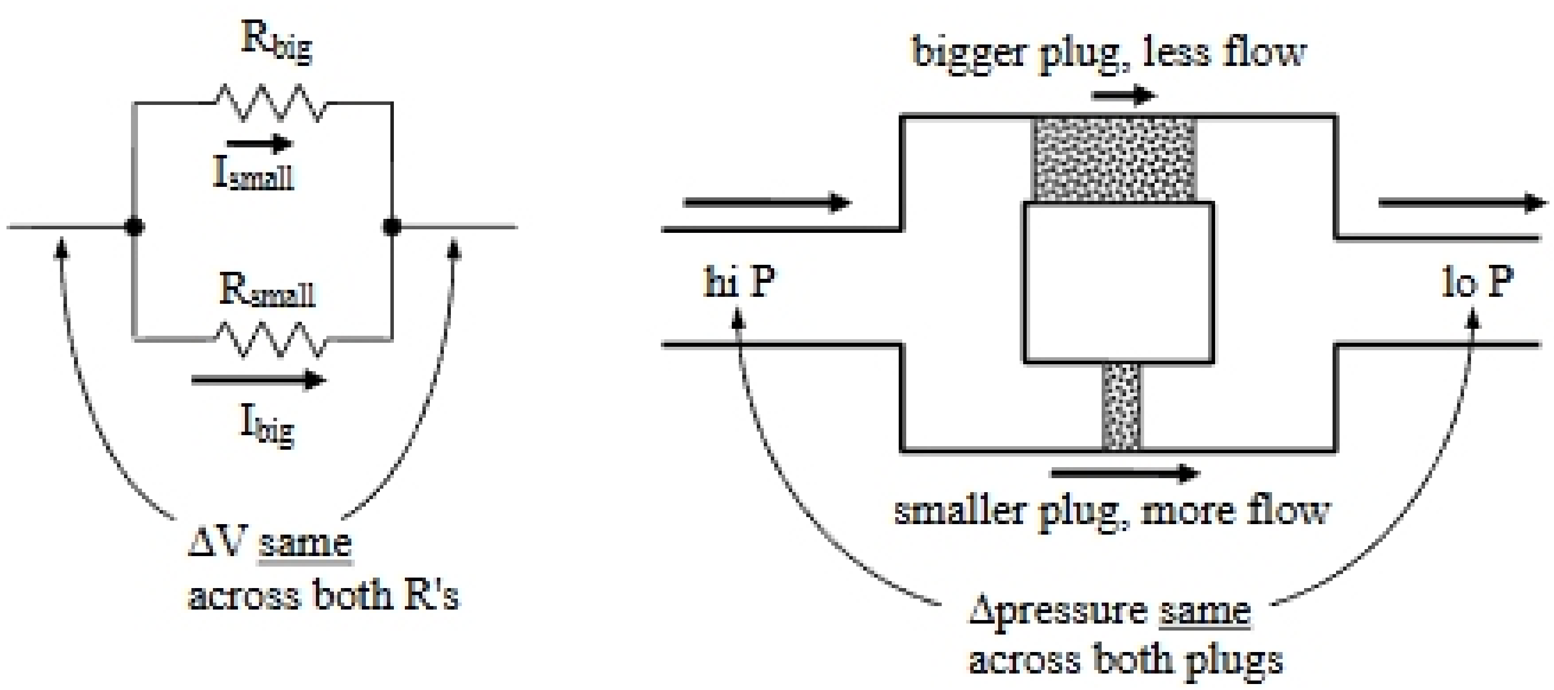
- The <u>current</u> is the <u>same</u> for resistors <u>in series</u>. Current is not "used up".



Think of the water pipe analogy: two gravel plugs in series, same flow (same gal/min) through both plugs (assuming no leaks or bubble in the pipe)



- Adding another resistor in series always increases the total resistance.
- The <u>voltage</u> difference across each resistor is the <u>same</u> for resistors <u>in parallel</u>.

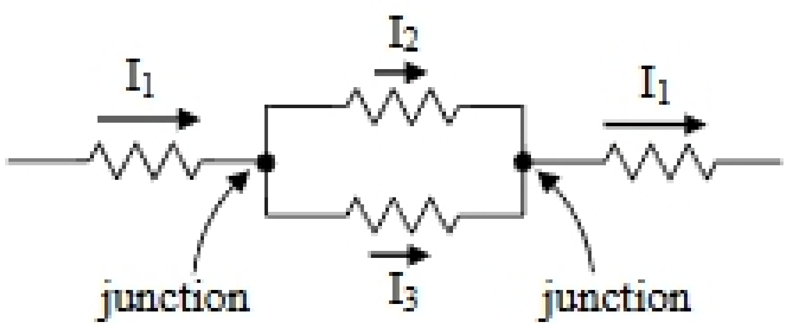


Both resistors in parallel have the same $\Delta V = V = I_{big} R_{small} = I_{small} R_{big}$.

- Adding another R in parallel always decreases the total resistance. Like adding another pipe along side the original pipe ⇒ allows more flow ⇒ smaller total resistance

Kirchhoff's two rules for analyzing circuits (*Kirchhoff* is really spelled that way: 2 h's, 2 f's)

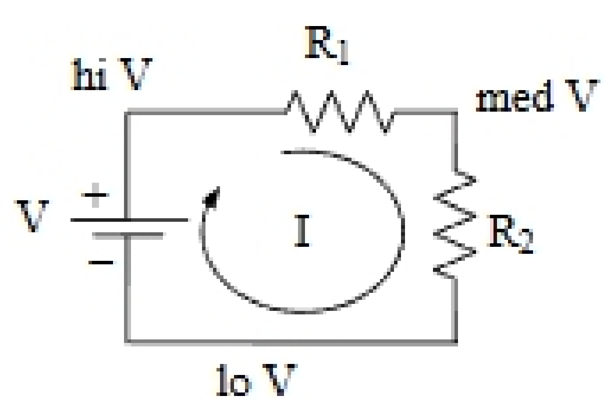
Kirchhoff's Current Rule (also called the Junction Rule)
The total current into any junction = total current out
(*junction* = place where 3 or more wires meet)

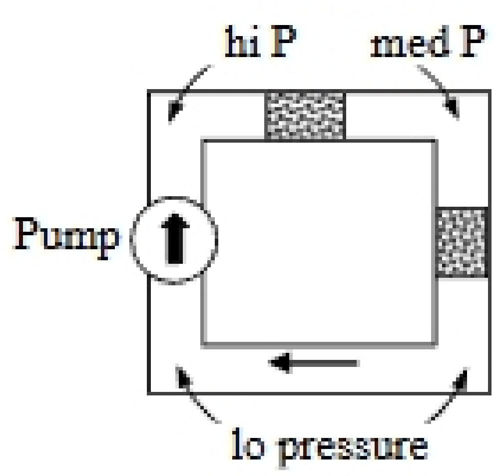


$$I_1 = I_2 + I_3$$

This is also called Conservation of Current. In steady-state, the charge is not building up anywhere, it is just flowing along at a steady rate. So the current into any portion of the circuit must equal the current coming out of that portion, otherwise charge would be building up in that part of the circuit.

Kirchhoff's Voltage Rule (also called the Loop Rule)
The sum of the voltage rises around any complete loop in a circuit = sum of the voltage drops around the same loop. Voltage rises and drops must sum to zero, since we must return to the same voltage after one complete loop.





$$\underbrace{V}_{\text{rise}} = \underbrace{IR_1}_{\text{fall}} + \underbrace{IR_2}_{\text{fall}} = I(R_1 + R_2) \Rightarrow I = \frac{V}{R_1 + R_2}$$

Remember: voltage is a kind of "electrical pressure" or "electrical height" . If you go around a complete circuit and return to the same place, you are back at the same pressure (or height). So rises must equal drops.