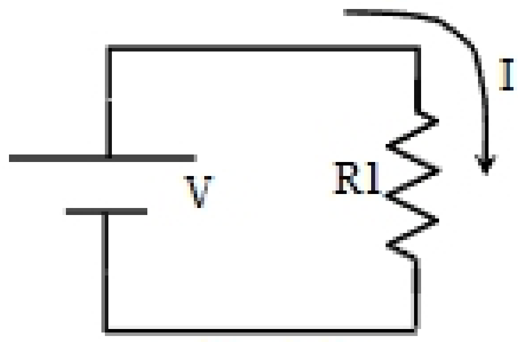
Electrical Circuits:

Many real world electronic devices are just collections of wires, resistors, capacitors, and batteries, forming circuits that do something (flashlights, toasters, blowdryers, radios, amplifiers,...) It's important to understand (and predict) the currents and voltages in such circuits. E.g., consider first a simple "flashlight circuit":

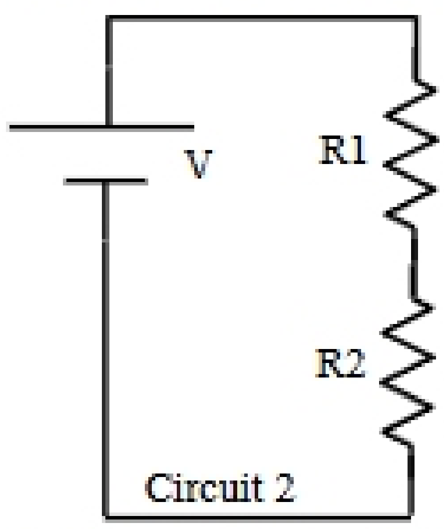


Circuit 1

The "R" here might represent the resistance of the flashlight bulb.

Here, V=I*R1, or I = V*(1/R1)

Now consider a slightly more complicated circuit:

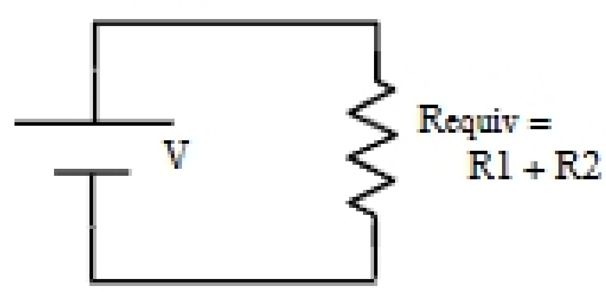


These resistors are in *series*.

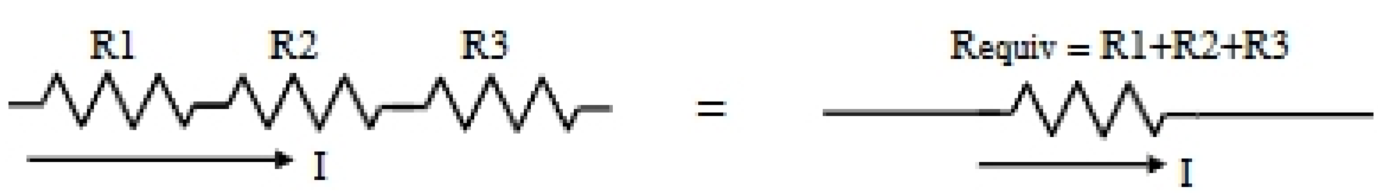
There will be a voltage drop V1= I*R1 across the first resistor, and V2 = I*R2 across the second.
The TOTAL voltage drop from top to bottom is V = V1+V2 = I*(R1+R2)

The resistances simply add up!

In other words, this circuit is essentially *equivalent* to the following simpler circuit:



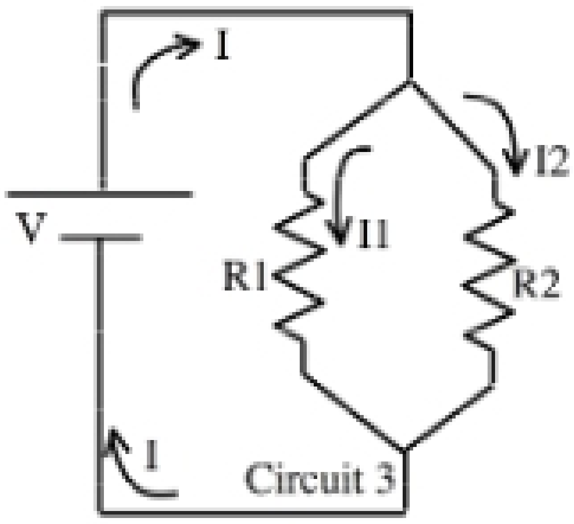
Similarly, for many resistors in series: $\boxed{R_{\text{equiv}} = R_1 + R_2 + R_3 + \ldots}$



Important: the current I is the SAME through each of these series resistors. (What goes in must come out: conservation of charge! This is not an approximation of any kind, it's exactly true)

However, that doesn't mean the current I in circuit 1 is the same as the current in circuit 2. Those are *different* circuits....

Here's a different circuit. We say R1 and R2 are "in parallel":



This time, the current I is NOT necessarily the same through R1 and R2.

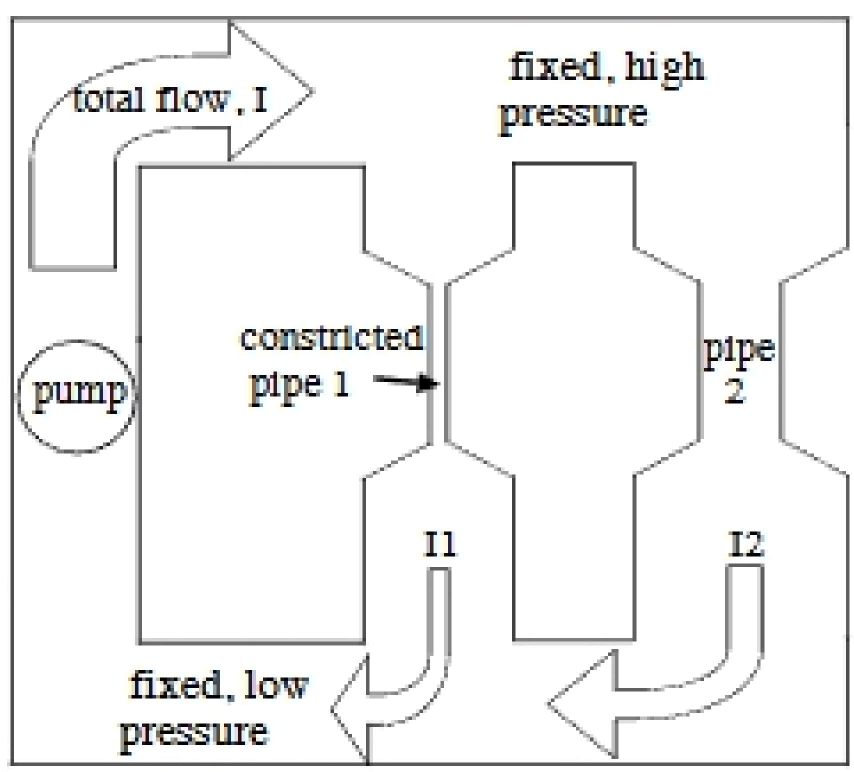
The current divides up: I1 goes left, I2 goes right.

(Conservation of charge, however, does tell us that I = I1 + I2, can you see why that is? Charges can't be created or destroyed here - they must go SOMEWHERE, and current just "counts the charges that flow by/sec")

It *also* says I at the bottom (going into the battery) is exactly the same as I at the top (leaving the battery)

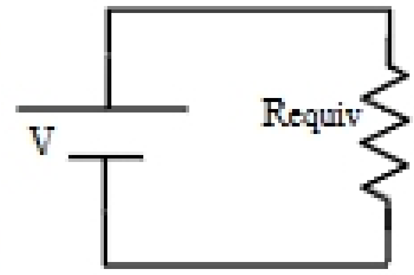
Note: *the voltage across R1 is exactly the same as the voltage across R2!* This is an important point, stare at the picture and try to understand why. Think of this as two different ski runs. Both have the same top and bottom (the same *height*, the same *voltage*), but they have different resistances, so different number of skiers/hour. (Different *currents* through each resistor)

Or, you might think of water flowing through pipes:
Here, the difference in pressure (like voltage difference) is exactly the same for both pipes (pressure at the top of either is identical, pressure at the bottom of either is identical, so the difference across either is identical) but the current through each will be different.



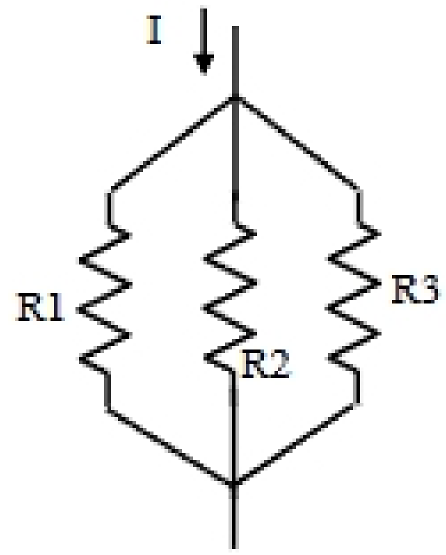
The total current is just the sum of the two currents, I = I1+I2

The previous parallel circuit (#3) is essentially *equivalent* to the following simpler circuit:



In this situation (resistors "in parallel") I claim

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

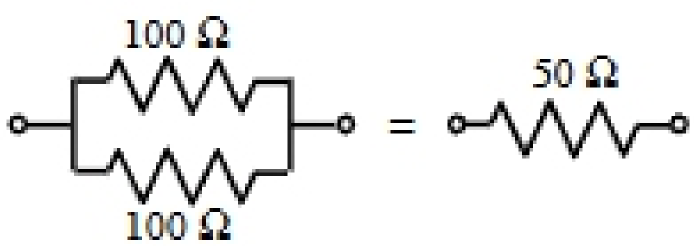


We can prove it mathematically (if you're interested):
It comes from the fact that $I = V / R_{\text{equiv}}$,
but conservation of charge says

$$I = I_1 + I_2 + I_3 + \ldots = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots$$

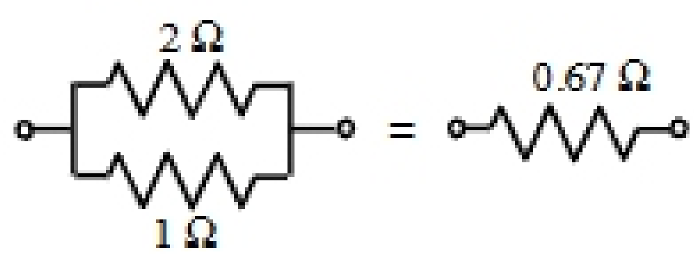
$$= V * \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \right) = V / R_{\text{equiv}}$$

Examples of equivalent resistances:



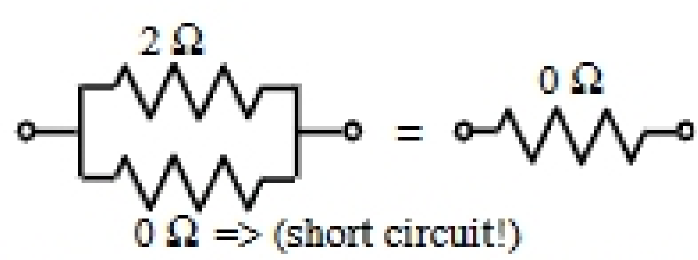
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{100\Omega} + \frac{1}{100\Omega} = .02\ \Omega^{-1},$$

$$i.e.\ \ R_{\text{equivalent}} = 1/(.02\ \Omega^{-1}) = 50\Omega$$



$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{2\Omega} + \frac{1}{1\Omega} = 1.5\ \Omega^{-1},$$

$$i.e.\ \ R_{\text{equivalent}} = 1/(1.5\ \Omega^{-1}) = 0.67\Omega$$



$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{2\Omega} + \frac{1}{0\Omega} = (0.5 + \infty)\ \Omega^{-1} = \infty\ \Omega^{-1},$$

$$i.e.\ \ R_{\text{equivalent}} = 1/(\infty\ \Omega^{-1}) = 0.\ \Omega$$

(The last is a short circuit, 0 resistance. All the current is happy to flow through the 0 Ω side!)
Note that R_Equiv *always* comes out *less* than any of the individual parallel R's. That means, if there are two (or more) ways for the current to go, there is LESS overall resistance to flow.

(More ways for current to flow makes it *easier* for the current to flow. More ski runs at a resort means you can get *more* people skiing: more current, less *overall resistance*.)