# THE HEMI-CUBE
# A RADIOSITY SOLUTION FOR COMPLEX ENVIRONMENTS

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ABSTRACT

This paper presents a comprehensive method to calculate object to object diffuse reflections within complex environments containing hidden surfaces and shadows. In essence, each object in the environment is treated as a secondary light source. The method provides an accurate representation of the "diffuse" and "ambient" terms found in typical image synthesis algorithms. The phenomena of "color bleeding" from one surface to another, shading within shadow envelopes, and penumbras along shadow boundaries are accurately reproduced. Additional advantages result because computations are independent of viewer position. This allows the efficient rendering of multiple views of the same scene for dynamic sequences. Light sources can be modulated and object reflectivities can be changed, with minimal extra computation. The procedures extend the radiosity method beyond the bounds previously imposed.

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General Terms: Algorithms

Additional Keywords and Phrases: Radiosity, diffuse reflections, hidden surface, form-factors, depth buffer.

INTRODUCTION

The representation of a realistic image of both actual and imagined scenes has been the goal of artists and scholars for centuries. The invention of the camera allowed the photographer to mechanically record the light passing through a lens and focused onto a piece of film, thus producing a realistic image. Today, within the field of computer graphics, one aspect of current research has been the attempt to produce realistic images of non-existent scenes. This is accomplished by simulating the distribution of light energy given a geometric and physical description of the environment. One major difficulty has been the correct simulation of the global illumination effects.



Light leaving an object surface originates from the surface by direct emission, as from a light source, or by the reflection or transmission of incident light. The incident light on a surface can arrive directly from a light source (along a direct line of sight) or indirectly by generally complex intermediate reflections and transmissions from other surfaces within the environment. Previously these secondary light "sources" have been ignored in computer graphics image generation algorithms. The summation of these sources have usually been approximated by an added constant term, referred to as the ambient component. [9] Some aspect of the global illumination has been achieved by the addition of a specular component found in ray tracing algorithms [13]. In essence, rays are traced only in the mirror reflection or transmission directions, point sampling the environment at the specific surface intersections. Ray tracing with

The FORM-FACTOR

The form-factor specifies the fraction of the energy leaving one surface which lands on another. By definition the sum of all the form-factors from a particular point or patch is equal to unity. Previous derivations do not account for occluding surfaces, and have therefore not reproduced shadows and penumbras.

The geometric terms in the form-factor derivation are illustrated in Figure 2. For non-occluded environments the form-factor for one differential area to another is given by:

$$F_{dAi\,dAj} = \frac{\cos\phi_i \cos\phi_j}{\pi r^2} \qquad (3)$$

By integrating over area j, the form-factor from a finite area (or patch) to a differential area can be expressed:

$$F_{dAi\,Aj} = \int_{A_j} \frac{\cos\phi_i \cos\phi_j}{\pi r^2}\, dAj \qquad (4)$$

The form-factor between finite surfaces (patches) is defined as the area average and is thus:

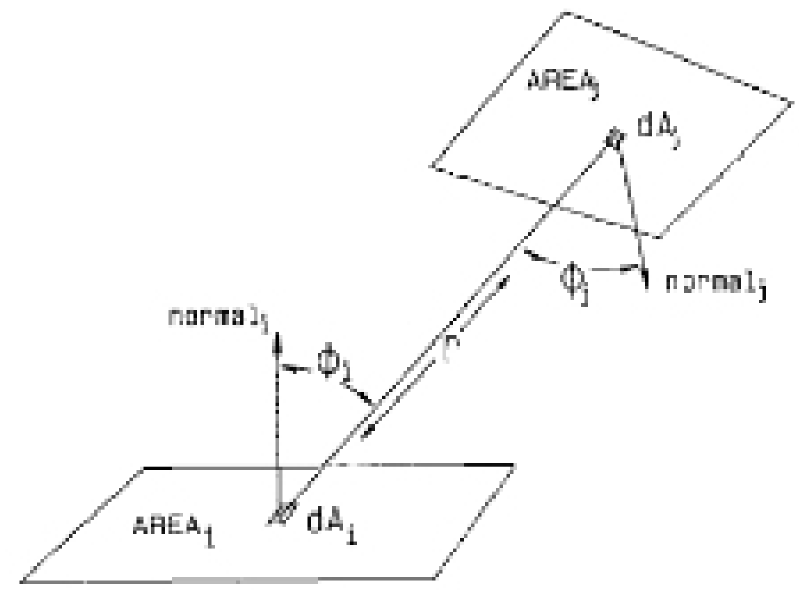
$$F_{Ai\,Aj} = \frac{1}{Ai} \int_{Ai}\int_{Aj} \frac{\cos\phi_i \cos\phi_j}{\pi r^2}\, dAj\, dAi \qquad (5)$$

This expression for the form-factor does not account for the possibility of occluding objects hiding all or part of one patch from another. There is, therefore, a missing term within the integrand if hidden surfaces are to be accounted for.

$$F_{Ai\,Aj} = \frac{1}{Ai} \int_{Ai}\int_{Aj} \frac{\cos\phi_i \cos\phi_j}{\pi r^2}\, \mathrm{HID}\, dAj\, dAi \qquad (6)$$

The function (HID) takes on a value of one or zero depending on whether differential area i can "see" differential area j. The HID function has the effect of producing the projection of area j visible from differential area i. It is the solution for this double area integral (6), which must be found to solve for radiosities in any non-convex environment.

In the past this double area integral has proven difficult to solve analytically for general applications. Form-factors between specific shapes and orientations, such as parallel rectanglar plates or circular disks, have been solved and tabulated [10]. An area integral, which is a double integral itself, can be




FORM-FACTOR GEOMETRY

Figure 2

transformed via Stoke's theorem into a single contour integral which can then be evaluated [5] [11]. For non-occluded environments the double area integral can be transformed into a double contour integral. A more general approach is needed to handle complex geometries.

Numerical techniques can provide a more efficient means to compute form-factors for general complex environments. Starting from a geometric analog to the analytic derivation, a numerical method is outlined to approximate the patch to patch form-factors which includes the hidden surface effects.

If the distance between the two patches is large compared to their size, and they are not partially occluded from one another, it can be seen that the integrand of the inner integral remains almost constant. In that case the effect of the outer integral is simply a multiplication by one and finding a solution to the inner integral will provide a good approximation for the form-factor. If the patches are close together relative to their size, or there is partial occlusion, the patches can be subdivided into smaller patches and the single integral equation approximation can still be used.

The form-factor from patch to patch is approximated with the differential area to finite area equation (4) by using the center point of patch i to represent the average position of patch i. Each patch has as its "view" of the environment, the hemisphere surrounding its normal.

A geometric analog for the form-factor integral was developed by Nusselt [10] and has been used to obtain form-factors by both photography and planimetry. (Figure 3). For a finite area, the form-factor is equivalent to the fraction of the circle (which is the base of the hemisphere) covered by projecting the area onto the hemisphere and then orthographically down onto the circle.