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Problem Solving Strategies Part IV

Working Backwards. Start with the final outcome of the problem and see if you can trace out the steps which get you there.

Example 1. See if you can devise a strategy which allows you to win the following game every time. 15 coins are placed on a table. You and another player take turns removing coins from the table. At each turn you may choose to take 1,2, or 3 coins. The player who removes the last coin is the winner.

To solve the problem, play the game backwards first.

- If I win: last move is to remove 1,2, or 3 coins from the table.
- This means my opponent left me with 1,2, or 3 coins.
- Now my opponent knows I will win if he leaves me with these choices. So he must have been forced to leave this many. That means he had to have started the move with 4 coins.
- So on my next to last move, I must leave my opponent with 4 coins.
- Now if I leave him with four coins on this move, I must have started with 5,6, or 7.
- I need to force opponent to leave me with 5,6, or 7 coins on the preceeding move.
- The only way to be sure that he does this is to leave him with 8 coins.
- To leave him 8 coins, I must have started with 9,10, or 11 coins.
- To force him to leave 9,10, or 11, I must have left him with 12.
- To leave him with 12, I must start the game and take three coins.

From this, I guess that my strategy is to always start the game, take 3 on the first move, leave him with 8 on the next move, and 4 on the next.

To see that this strategy works, I play the game forwards now.

1. I start the game with 15 and take out 3. This leaves the opponent with 12.
2. Opponent takes 1,2,or 3 from 12 leaving me with 9,10, or 11.
3. Since I'm left with 9,10, or 11 I leave the opponent with 8.
4. Opponent takes 1,2,or 3 from 8 and leaves me 5,6,or 7.
5. Since I'm left with 5,6,or 7, I leave the opponent with 4.
6. Opponent takes 1,2, or 3 from 4 and leaves 1,2, or 3.
7. I take 1,2,or 3 for the win.

Eliminate Possibilities. If you can't solve a problem immediately, try to figure out what can't happen first.

Example 2. Beth, Jane, and Mitzi play on the basketball team. Their positions are forward, center, and guard. Given the following information, determine who plays each position:

- Suppose that Beth and the guard bought a milkshake for Mitzi.
- Beth is not the forward.

The Pigeonhole Principle. Suppose $m > n$. If there are m pigeons and n pigeon holes then there will be at least one pigeonhole with more than one pigeon.

The idea of this principle is the following. Suppose I have 6 pigeons and five pigeonholes. Then *at least* one pigeon hole will have more than one pigeon in it. We can also say, for example, that if there are 100 hotel rooms booked for 104 guests then at least one of these rooms will have more than 1 guest in it.

Example 3. A box contains 30 blocks: 10 red, 10 blue, and 10 green. You are blindfolded and asked to draw the smallest possible number of blocks from the box so that you have at least two blocks of the same color. How many should you pick?

Problems: Work in a group of 3-4 to solve the following three practice problems.

1. Moe, Joe, and Hiram are brothers. One day, in some haste, they left home with each one wearing the hat and coat of one of the others. Joe was wearing Moe's coat and Hiram's hat. Whose hat and coat was each one wearing?

2. Can you devise a strategy which will always win the following game? The first player marks down 1,2,3, or 4 tallies on a sheet of paper. The second player then adds to this by marking down 1,2,3, or 4 more tallies. The first player to exceed a total of 30 loses the game.