

## More Problem Solving Strategies: Part I

One of your most important tasks as an elementary educator will be to provide students with many “tools” with which to solve mathematics problem. In the context of Polya’s principles, this corresponds to giving students lots of options in the *devise a plan stage*. You have already seen the following important problem solving tools:

- Guess and check
- Diagrams
- Lists.

The rest of chapter 1 will be devoted to expanding this toolbox.

**Look for a Pattern.** Look at an ordered sequence of particular examples from the problem. Analyze the results and find a pattern which will tell you how to solve the problem.

Some examples:

- 1, 4, 7, 10, 13, \_\_\_\_, \_\_\_\_, \_\_\_\_.

1. Understand: We know the first 5 terms. We must find the next three terms.
2. Plan: Analyze the first 5 terms. Find the pattern and use it to find the next three terms.
3. Execute: Well,  $1 + 3 = 4$ ,  $4 + 3 = 7$ ,  $7 + 3 = 10$ , and  $10 + 3 = 13$ . So I guess that the pattern is the next term is the previous term plus three. So the answer is 16,18,21.
4. Review: To find the pattern we looked at previous term and tried to understand how it was related to the next term. This allowed us to discover the pattern and write the remaining terms.

- 19, 20, 22, 25, 29, \_\_\_\_, \_\_\_\_, \_\_\_\_ . Answer: 34, 40, 47

- 1, 4, 9, 16, 25, \_\_\_\_, \_\_\_\_, \_\_\_\_ . Answer:  $6^2 = 36$ ,  $7^2 = 49$ ,  $8^2 = 64$ . Can you guess what the tenth term is?

- 1, 1, 2, 3, 5, \_\_\_\_, \_\_\_\_, \_\_\_\_ . Answer: 8, 13, 21. This is a special sequence called **the Fibonacci sequence**. The terms of this sequence are called **Fibonacci numbers**. We use the notation

$F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, \dots, F_n =$  nth Fibonacci number and so on. Notice that

$$F_3 = 2 = 1 + 1 = F_2 + F_1$$

$$F_4 = 3 = 2 + 1 = F_3 + F_2$$

$$F_5 = 5 = 3 + 2 = F_4 + F_3$$

$\vdots$

$$F_{N+1} = F_N + F_{N-1}$$

The Fibonacci numbers have been studied for hundreds of years and have many interesting applications and properties.

**Note:** Sometimes patterns are very hard to see just by inspection. In cases like this it is sometimes helpful to make a table. See page 23 in the text for an example like this.

**A Special Pattern: Pascal's Triangle** A very famous pattern is the array of numbers called Pascal's triangle. (See description and construction on page 27.) In addition to being useful as a problem solving tool, the triangle itself contains many interesting patterns many of which are basic enough for elementary students to discover on their own. Thus it can be an excellent tool for students to explore and experiment with the properties of numbers.