1443-501 Spring 2002
Lecture #12

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1. Motion of a System of Particles
2. Angular Displacement, Velocity, and Acceleration
3. Angular Kinematics Under Constant Angular Acceleration
4. Relationship Between Angular and Linear Quantities

Remember the mid-term exam on Wednesday, Mar. 13. Will cover Chapters 1-10.

Today's Homework Assignment is the Homework #4!!!

Motion of a Group of Particles

We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass M is preserved, th... of...

Velocity of the system

$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum m_i \vec{r}_i\right) = \frac{1}{M}\sum m_i \frac{d\vec{r}_i}{dt} = \frac{\sum m_i \vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{tot} = M\vec{v}_{CM} = M\frac{\sum m_i \vec{v}_i}{M} = \sum m_i \vec{v}_i = \sum \vec{p}_i$$

Acceleration of the system

$$\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum m_i \vec{v}_i\right) = \frac{1}{M}\sum m_i \frac{d\vec{v}_i}{dt} = \frac{\sum m_i \vec{a}_i}{M}$$

External force exerting on the system

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} = \sum m_i \vec{a}_i = \frac{d\vec{p}_{tot}}{dt}$$

What about the internal forces?

If net external force is 0

$$\sum \vec{F}_{ext} = 0 = \frac{d\vec{p}_{tot}}{dt}; \quad \vec{p}_{tot} = \text{const}$$

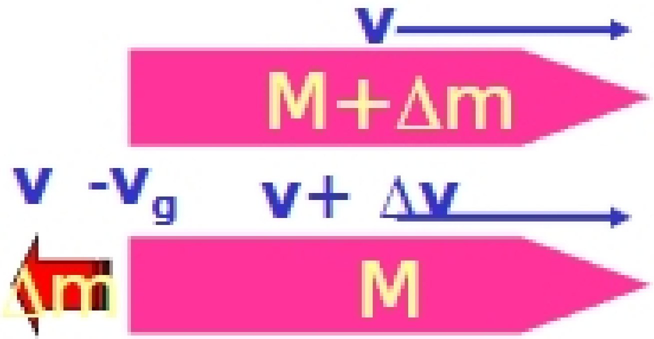
System's momentum is conserved.

Rocket Propulsion

What is the biggest difference between ordinary vehicles and a rocket?

The force that gives propulsion for normal vehicles is the friction between the surface of the road and the tire. The system in this case consists of the tire and the surface of the road. Newton's 3rd law and the momentum conservation of an isolated system.

Since there is no road to push against, the rockets obtain propulsion from momentum conservation in the system consists of the rocket and gas from burnt fuel.



Initial momentum before burning fuel

$$\vec{p} = (M + \Delta m)\vec{v}$$

Final momentum after burning fuel and ejecting the gas

$$\vec{p} = M(\vec{v} + \Delta\vec{v}) + \Delta m(\vec{v} - \vec{v}_g)$$

From momentum conservation

$$M(\vec{v} + \Delta\vec{v}) + \Delta m(\vec{v} - \vec{v}_g) = M\vec{v} + \Delta m\vec{v}; \quad M\Delta\vec{v} = \Delta m\vec{v}_g$$

Since dm is the same as –dM, one can obtain

$$d\vec{v} = \frac{dm\vec{v}_g}{M}; \quad \int_i^f d\vec{v} = \vec{v}_f - \vec{v}_i = -\vec{v}_g \int_i^f \frac{dM}{M} = \vec{v}_g \ln\left[\frac{M_i}{M_f}\right]$$

Thrust is the force exerted on the rocket by the ejected gas

$$Thurst = M\frac{dv}{dt} = \left|v_g \frac{dM}{dt}\right|$$