Lecture 21 — Identical Particles
Chapter 6, Wednesday February 27th

- Review of Lecture 19
- Calculating partition function for identical particles
- Dilute and dense gases
- Identical particles on a lattice
- Spin and rotation in diatomic molecules (if time)

Reading: All of chapter 6 (pages 128 - 142)
Assigned problems, Ch. 6: 2, 4*, 6, 8, (+1)
Homework 6 due on Friday 29th
1 more homework before spring break (Fri.)
Exam 2 on Wed. after spring break

Bosons

$$\psi_{2,\mathrm{Bose}}(x_1,x_2)=\frac{1}{\sqrt{2}}\left\{\phi_i(x_1)\phi_j(x_2)+\phi_i(x_2)\phi_j(x_1)\right\}=\psi_{2,\mathrm{Bose}}(x_2,x_1)$$

$$\begin{aligned}\psi_{3,\mathrm{Bose}}(x_1,x_2,x_2)=&\phi_i(x_1)\phi_j(x_2)\phi_k(x_3)+\phi_i(x_2)\phi_j(x_1)\phi_k(x_3)\\&+\phi_i(x_2)\phi_j(x_3)\phi_k(x_1)+\phi_i(x_3)\phi_j(x_2)\phi_k(x_1)\\&+\phi_i(x_1)\phi_j(x_1)\phi_k(x_2)+\phi_i(x_1)\phi_j(x_3)\phi_k(x_2)\end{aligned}$$

- Wavefunction symmetric with respect to exchange. There are N! terms.
- Another way to describe an N particle system:

$$\psi_i=\left|n_1,n_2,n_3,\cdots\right\rangle$$

$$E_i=n_1\varepsilon_1+n_2\varepsilon_2+n_3\varepsilon_3+\cdots$$

- The set of numbers, n_i, represent the occupation numbers associated with each single-particle state with wavefunction $\phi_{i.}$
- For bosons, occupation numbers can be zero or ANY positive integer.

Fermions

$$\psi_{2,\text{Fermi}}(x_1,x_2)=\frac{1}{\sqrt{2}}\left\{\phi_i(x_1)\phi_j(x_2)-\phi_i(x_2)\phi_j(x_1)\right\}=-\psi_{2,\text{Fermi}}(x_2,x_1)$$

- Alternatively the N particle wavefunction can be written as the determinant of a matrix, e.g.:

$$\psi_{3,\text{Fermi}}(x_1,x_2,x_3)=\begin{vmatrix}\phi_i(x_1) & \phi_j(x_1) & \phi_k(x_1)\\ \phi_i(x_2) & \phi_j(x_2) & \phi_k(x_2)\\ \phi_i(x_3) & \phi_j(x_3) & \phi_k(x_3)\end{vmatrix}$$

- The determinant of such a matrix has certain crucial properties:
  1. It changes sign if you switch any two labels, i.e. any two rows.

     It is **antisymmetric** with respect to exchange

  2. It is ZERO if any two columns are the same.
- Thus, you cannot put two Fermions in the same single-particle state!