$\boxed{\text{Section 6.1}}$ Integration by Parts

$$\int \ln(x)\,dx = ?$$

We will develop a formula from the product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$d(uv) = u\,dv + v\,du$$

$$u\,dv = d(uv) - v\,du$$

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

<u>Integration by Parts Formula</u>

If u and v are continuous functions then

$$\int u\,dv = uv - \int v\,du$$

Example: compute $\int \ln(x)\,dx$

$$u = \ln(x) \qquad dv = dx$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int dx$$

$$v = x$$

$$\int du = \frac{1}{\cancel{x}}\,dx$$

$$\int u\,dv = uv - \int v\,dv$$

$$= x\ln(x) - \int dx$$

$$\int \ln(x)\,dx = x\ln(x) - x + C$$

$$\frac{d}{dx}\left[x\ln(x) - x + C\right]$$

$$\ln(x) + 1 - 1 = \ln(x)$$

Example: $\int xe^{-3x}\,dx$ $\qquad u=x \qquad dv=\int e^{-3x}dx$

$\qquad\qquad\qquad\qquad du=dx \qquad v=-\frac{1}{3}x^{-3x}$

$$\int u\,dv = uv-\int v\,du$$

$$=-\frac{1}{3}xe^{-3x}+\frac{1}{3}\int e^{-3x}dx$$

$$\int xe^{-3x}dx=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}+C$$

Example: Suppose we had made the following choices for u and dv in the previous problem?

$$\int xe^{-3x}dx$$

$u=e^{-3x} \qquad dv=x\,dx$

$du=-3e^{-3x}dx \qquad v=\frac{1}{2}x^2$

$$=\frac{1}{2}x^2e^{-3x}-\frac{1}{2}\int x^2e^{-3x}dx$$

This integral is more complex to compute than the original integral

Example: $\int x^2 e^{-3x}\,dx$

$u = x^2 \qquad dv = e^{-3x}\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{3}e^{-3x}$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int xe^{-3x}\,dx$

$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$

$= e^{-3x}\left[\frac{1}{3}x^2 - \frac{2}{9}x - \frac{2}{27}\right] + C$

Example: $\int \frac{x}{(x-2)^3}\,dx = \int x(x-2)^{-3}\,dx$

$u = x \qquad dv = \int (x-2)^{-3}\,dx$

$du = dx \qquad v = -\frac{1}{2}(x-2)^{-2}$

$= -\frac{1}{2}x(x-2)^{-2} + \frac{1}{2}\int (x-2)^{-2}\,dx$

$= -\frac{1}{2}x(x-2)^{-2} + -\frac{1}{2}(x-2)^{-1} + C$