

PH314 Midterm #2

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$c = 3 \times 10^{10} \text{ cm s}^{-1}$	speed of light
$G = 6.67 \times 10^{-8} \text{ gm}^{-1} \text{ cm}^3 \text{ s}^{-2}$	universal constant of gravitation
$M_{\odot} = 2 \times 10^{33} \text{ gm}$	Solar mass
$L_{\odot} = 3.9 \times 10^{33} \text{ ergs s}^{-1}$	Solar luminosity
$M_{\text{earth}} = 6 \times 10^{27} \text{ gm}$	mass of the Earth
$m_p = 1.67 \times 10^{-24} \text{ gm}$	mass of the proton
$k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$	Boltzmann's constant
$h = 6.63 \times 10^{-27} \text{ gm cm}^2 \text{ s}^{-1}$	Plank's constant
$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$	Stefan-Boltzmann constant

Please write your name on each piece of paper used. There is partial credit so try to answer all problems. I care more about the derivation of the formulae than the numerical answer. Each problem is worth the same amount even if there is more than one part to the question.

1. Star S2 orbits the center of the Galaxy in 15.2 years. The orbital semi-major axis is 950 AU (5 light-days). What is the mass of the black hole at the center of the Galaxy? Hint: Use Kepler's relation written as $P^2[M_{\text{BH}} + M_{\star}]/M_{\odot} = a^3$ where P is the period in years, "a" is the semi-major axis in AU, M_{BH} is the mass of the black hole, and M_{\star} is the mass of the star orbiting the black hole. Leave the units in solar masses which makes the calculation easier.

2. We know that a black hole will evaporate by Hawking radiation. The luminosity of the black hole is $L_{\text{bh}} = hc^6/(30720\pi^2 G^2 M_{\text{bh}}^2)$ or $L_{\text{bh}} = 3.6 \times 10^{45} \text{ gm}^2 \times M_{\text{bh}}^{-2} \text{ erg/s}$. Argue that the evaporation time for a black hole is roughly $t_{\text{evap}} \sim M_{\text{bh}} c^2 / L_{\text{bh}}$. Because the evaporation accelerates as the black hole radiates away its mass, the actual formula is $t_{\text{evap}} \sim M_{\text{bh}} c^2 / (3L_{\text{bh}})$. Calculate the evaporation time for a black hole with a mass of $2 \times 10^{14} \text{ gm}$ and express it in billions of years. Draw appropriate astronomical conclusions about much more massive and much less massive black holes. Can dark matter be these mini black holes with masses of $2 \times 10^{14} \text{ gm}$?

3. We are about 8.5 kpc from the center of the Galaxy and the other side of the Galaxy is about 20 kpc away. How bright would a Cepheid be at the distance of 20 kpc? Assume that the Cepheid has an absolute magnitude of $M = -5.5$. Remember that the absolute magnitude is the brightness of a star at $d = 10 \text{ pc}$. Hint: Remember that $m = -2.5 \log_{10}(L/(4\pi d^2)) + \text{constant}$. Derive the formula $m - M = 5 \log_{10}(d) - 5$, and solve for m . We can't see these Cepheids at the other side of our Galaxy. Why not?

4. R. J. Trumpler was the first astronomer to show that there was dust in the Galaxy. He did this by assuming that all open clusters of stars in the disk of the Galaxy have the same physical diameter. If D is the real diameter of the cluster and R is the distance to the

cluster, draw a picture showing that the angular size of the cluster is $\theta = (D/R)$. If we assume that all clusters have the same luminosity L then the observed flux is $f=L/4\pi R^2$. Show that if you plot the \log_{10} of the solid angle of the cluster (solid angle = θ^2) on the x-axis, and the \log_{10} of the observed flux of the cluster on the y-axis, you get a straight line. What is the slope? Draw this graph assuming no dust and show where the near clusters and the distant clusters are located. Now assume that there is dust everywhere. Explain how the relationship changes, by showing the effects on the graph.

5. We have discussed three observations that show the presence of dark matter. Give two cases and explain qualitatively why they show the presence of dark matter.

6. Show that the number of objects brighter than a given flux f_0 goes as $f_0^{-3/2}$, that is, $N_L(f > f_0) = A \times f_0^{-3/2}$ where $A = n(L)L^{3/2}/(3 \times (4\pi)^{1/2})$. Assume the number density of the objects, $n(L)$ is constant throughout the volume. Remember that flux = $L/4\pi r^2$ and calculate the total number as $N_L = n(L) \times \text{volume}$.

7. The Tully-Fisher relation says that a galaxy's total luminosity is related to the rotation velocity as $L_{\text{gal}} \propto v^4$ (where \propto means "proportional"). Assume that a galaxy is a disk with constant mass density ρ , and that its mass is just the disk area times twice the height H of the galaxy disk. Show that the mass of the galaxy goes as $M_{\text{gal}} = 2H\pi R^2\rho$. Show that this implies the mass-to-light ratio for the galaxy, $M_{\text{gal}}/L_{\text{gal}}$ is independent of R and v . Hint: remember that centripetal force = gravitational force and show that $v^2 \propto M/R$.

8. (a) What is the name of our Galaxy? (b) What is the name of the other large galaxy in our local group of galaxies? (c) What is the name given to our local group of galaxies? (d) What is the largest structure of galaxies in the Universe?