## Example $P_{avg} = I_{rms}\Delta V_{rms} = \frac{(\Delta V_{rms})^2}{R} = I_{rms}^2 R$

- The household power in the US has an rms voltage of 120 V. A light bulb in an household circuit is rated 100 W (avg. power).

(a) What is the peak voltage of the household circuit?

$$\Delta V_{max} = \Delta V_{rms} \times \sqrt{2} = 120 \times \sqrt{2} = 169.7\ V$$

(b) What is the resistance of the light bulb at operating conditions?

$$P_{avg} = \frac{(\Delta V_{rms})^2}{R} \quad \Rightarrow R = \frac{(\Delta V_{rms})^2}{P_{avg}} \quad \Rightarrow R = \frac{(120)^2}{100} = 144\Omega$$

(c) What are the rms and peak currents in the light bulb when operating?

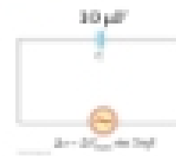
$$P_{avg} = I_{rms}\Delta V_{rms} \quad \Rightarrow I_{rms} = \frac{P_{avg}}{\Delta V_{rms}} \quad \Rightarrow I_{rms} = \frac{100}{120} = 0.833A$$

$$I_{max} = \sqrt{2} I_{rms} = 1.18\ A$$

## Example $\Delta v = 200\sin(2\pi f t);\ f = 60\ Hz$

- Pure resistor:


- Pure capacitor




Maximum voltage :
$\Delta V_{max} = 200\ V$

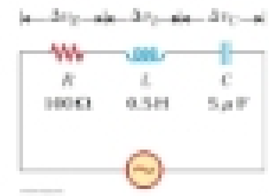
rms voltage
$\Delta V_{rms} = 200/\sqrt{2}$
$= 141\ V$

rms current
$I_{rms} = \Delta V_{rms} / R = 1.41A$

Capacitive reactance?
$X_c = \frac{1}{2\pi f C}$
$= \frac{1}{6.28 \times 60 \times 10 \times 10^{-6}}$
$= 265\Omega$

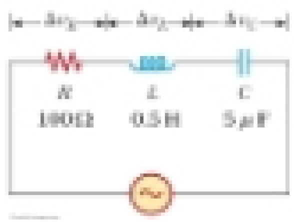
rms current?
$I_{rms} = \Delta V_{rms} / X_c$
$= 141/265$
$= 0.53A$

## Example $\Delta v = 200\sin(2\pi f t);\ f = 60\ Hz$




1. What is the impedance of the circuit?
2. Phase angle φ?
3. $I_{max}$?
4. Maximum voltage drop across each element?

## Example $\Delta v = 200\sin(2\pi f t);\ f = 60\ Hz$




(1) What is the impedance of the circuit?

$X_C = \frac{1}{2\pi f C}$
$= \frac{1}{6.28 \times 60 \times 5 \times 10^{-6}}$
$= 530\ \Omega$

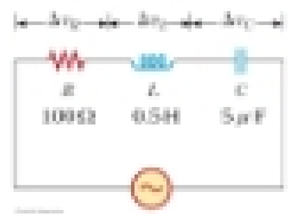
$X_L = 2\pi f L$
$= 6.28 \times 60 \times 0.5$
$= 188\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$
$= \sqrt{100^2 + (188 - 530)^2}$
$= 356\Omega$

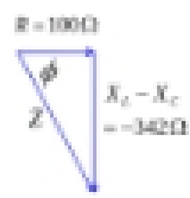
(2) Phase angle φ?

$\varphi = \tan^{-1}(\frac{X_L - X_C}{R})$
$= \tan^{-1}(\frac{188 - 530}{100})$
$= -73.7°$

## Example $\Delta v = 200\sin(2\pi f t);\ f = 60\ Hz$




Phasor diagram




(3) $I_{max}$?

$I_{max} = \frac{\Delta V_{max}}{Z} = \frac{200}{356}$
$= 0.56A$

(4) Maximum voltage drop across each element?

$\Delta V_{R,max} = I_{max} R = 56V$
$\Delta V_{C,max} = I_{max} X_C = 297\ V$
$\Delta V_{L,max} = I_{max} X_L = 105\ V$

$\Delta V_{R,max} + \Delta V_{L,max} + \Delta V_{C,max} \neq 200V$

## Problem Solving Strategy

- Calculate the inductive and capacitive reactances, $X_L$ and $X_C$

$$X_L = 2\pi f L;\quad X_C = \frac{1}{2\pi f C}$$

- Draw phasor diagram, find Z and the phase angle

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \tan(\phi) = \frac{X_L - X_C}{R}$$

## Example: Power in an AC Circuit

- Calculate the average power delivered in the previous example.

Solution: We have calculated

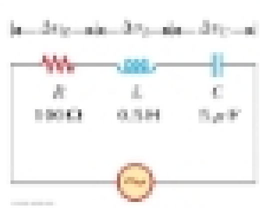
$\Delta V_{R,max} = 56\ V$

$I_{max} = \frac{\Delta V_{max}}{Z} = 0.562\ A$

Therefore, $\Delta V_{R,rms} = \frac{56}{\sqrt{2}} = 39.6V$

$I_{rms} = \frac{0.562}{\sqrt{2}} = 0.397\ A$

$P_{av} = I_{rms}\Delta V_{R,rms} = 15.7\ W$

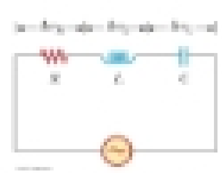



$\Delta v = 200\sin(2\pi f t);\ f = 60Hz$
$Z = \sqrt{R^2 + (X_L - X_C)^2} = 356\ \Omega$

Please verify
$P_{av} = I_{rms}\Delta V_{rms}\cos(\varphi) = 15.7\ W$

## Example

- Consider a RLC circuit. R=150 Ω, L=20 mH, $\Delta V_{rms}$=20 V, the frequency of the AC source if 796 Hz.



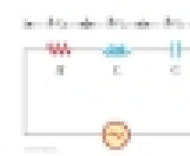
(a) Find the value of the capacitance for which the rms current in the circuit is a maximum.

Solution: The capacitance must be that the frequency of the AC source is the resonance frequency. Therefore,

$$f_0^2 = \frac{1}{4\pi^2 LC} \qquad \Rightarrow C = \frac{1}{4\pi^2 L \cdot f_0^2}$$

$$= \frac{1}{4\pi^2(20 \times 10^{-3}) \times (796)^2} = 2.0 \times 10^{-6}\ F$$

## Example



(b) What is the maximum rms current in the circuit?

Solution: At resonance, the total impendence of the circuit is just the resistance R. Therefore,

$$I_{rms} = \frac{\Delta V_{rms}}{Z} = \frac{\Delta V_{rms}}{R} = \frac{20}{150} = 0.133A$$

(c) What is the rms values of the voltage drops across the capacitor and inductor at resonance?

Solution: $X_L = 2\pi f L = 2\pi \times 796 \times (20 \times 10^{-3}) = 100\ \Omega$

$\Delta V_{rms}^L = I_{rms} \cdot X_L = 0.133 \times 100 = 13.3V$

$\Delta V_{rms}^C = \Delta V_{rms}^L$ at resonance

## Example

- A power station produces $\begin{cases}\Delta V_{rms} = \Delta V_1 = 4000\ V \\ I_{rms} = I_1 = 100\ A\end{cases}$
- Suppose the resistance of transmission line is $R = 30\Omega$. What is the power lost in transmission line?
- What is the power loss if the voltage is stepped up to $2.4 \times 10^5$ V?

## Example

$$\begin{cases}\Delta V_{rms} = \Delta V_1 = 4000\ V \\ I_{rms} = I_1 = 100\ A \\ R = 30\Omega\ \text{(resistance of transmission line)}\end{cases}$$

(1) Power generated:

$$P_{generated} = I_1\Delta V_1 = 4 \times 10^5\ W$$

(2) Power lost in transmission line:

$$P_{lost} = I_1^2 R = 100^2 \times 30 = 3 \times 10^5\ W$$

(3) Percentage of power lost:

$$\frac{P_{lost}}{P_{generated}} = \frac{3 \times 10^5}{4 \times 10^5} = 75\%\ !!!$$

## Example

$$\begin{cases}\Delta V_{rms} = \Delta V_1 = 4000\ V \\ I_{rms} = I_1 = 100\ A \\ R = 30\Omega\ \text{(resistance of transmission line)}\end{cases}$$

What is the power loss if the voltage is stepped up to $2.4 \times 10^5$ V?

New transmission current

$P_{generated} = I_2\Delta V_2 = I_1\Delta V_1$

$I_2 = I_1\Delta V_1 / \Delta V_2$
$= \frac{4 \times 10^5}{2.4 \times 10^5} = 1.67\ A$

$$\frac{P_{lost}}{P_{generated}} = \frac{I_2^2 R}{4 \times 10^5} = \frac{1.67^2 \times 30}{4 \times 10^5} = 0.02\%\ !!!$$

## 20.7 Energy stored in a magnetic field (in an inductor)

- An electric current produces a magnetic field (19.7, 19.9).
- Energy is stored in a magnetic field (just as energy is stored in an electric field)
- For an inductor with an inductance L and a current I passing through it, the energy stored in the magnetic field can also be viewed as energy stored in the inductor

$$E = \frac{1}{2} L \cdot I^2$$

- Compare with energy stored in a capacitor

$$E = \frac{1}{2} C \cdot (\Delta V)^2$$