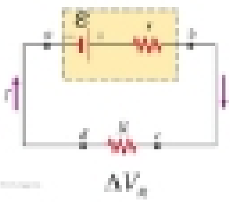
Example:

$$\begin{cases} \varepsilon = 1.5 \text{ V} \\ R = 2\Omega \\ \Delta V_R = 1.4 \text{ V} \end{cases}$$

- Given the circuit on the right, calculate the internal resistance of the battery.



Analysis: $\varepsilon = Ir + IR$, need to calculate current I

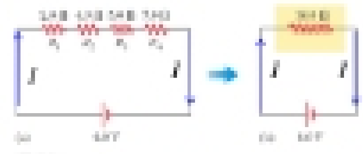
Solution: Current in the circuit:

$$I = \frac{\Delta V_R}{R} = \frac{1.4}{2.0} = 0.7 \text{ A}$$

$$\varepsilon = Ir + IR \quad \Rightarrow \quad Ir = \varepsilon - IR$$

$$\Rightarrow \ r = \frac{\varepsilon - \Delta V_R}{I} = \frac{1.5 - 1.4}{0.7} = 0.14\ \Omega$$

Example



- What is the power delivered to resistor R_1? $P = I^2 R_1$

(a) Calculate the current I through the resistors:

$$I = \frac{\Delta V}{R^{eq}} = \frac{6}{18} = 0.33 \text{ A}$$

Current is the same for all resistors in series.

This current is the same for all four resistors.

(b) Power consumed by resistor R_1:

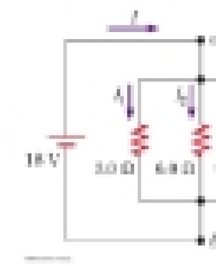
$$Power = I^2 R_1 = (0.33)^2 \times 2 = 0.22 \text{ W}$$

- What is the voltage drop ΔV_2 across resistor R_2?

From Ohm's law: $\Delta V_2 = IR_2 = 0.33 \times 5.0 = 1.67$ V

Example

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad R_{eq} = 1.64\Omega$$



- Power delivered to each resistor

$$P_1 = \frac{(\Delta V)^2}{R_1} = \frac{18^2}{3} = 108 \text{ W}$$

$$P_2 = \frac{(\Delta V)^2}{R_2} = \frac{18^2}{6} = 54 \text{ W}$$

$$P_3 = \frac{(\Delta V)^2}{R_3} = \frac{18^2}{9} = 36 \text{ W}$$

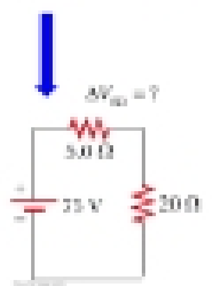
- Total power $P = P_1 + P_2 + P_3 = 198$ W
- Another way of calculating the total power $P = \frac{(\Delta V)^2}{R_{eq}} = 198$ W

Example

(2) What is the current passing through the 5 Ω resistor?

Solution: Current passing the 5 Ω resistor is the same as the total current:

$$I = \frac{75}{R_{eq}} = \frac{75}{25} = 3 \text{ A}$$



(3) What is the voltage drop across the 5 Ω resistor?

$$\Delta V_{5\Omega} = IR = 3 \times 5 = 15 \text{ V}$$

(4) What is the current passing through the 30 Ω resistor?
- try it yourself.

Kirchhoff's Rules

- Loop Rule: The sum of the potential differences across all the elements around any closed circuit loop must be zero

Loop abcda:

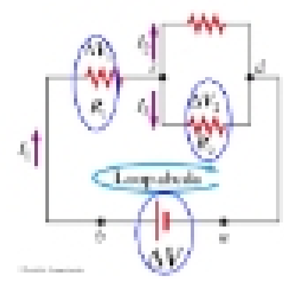
$$\Delta V + \Delta V_1 + \Delta V_2 = 0$$

$$\Delta V = V_b - V_a = \varepsilon$$

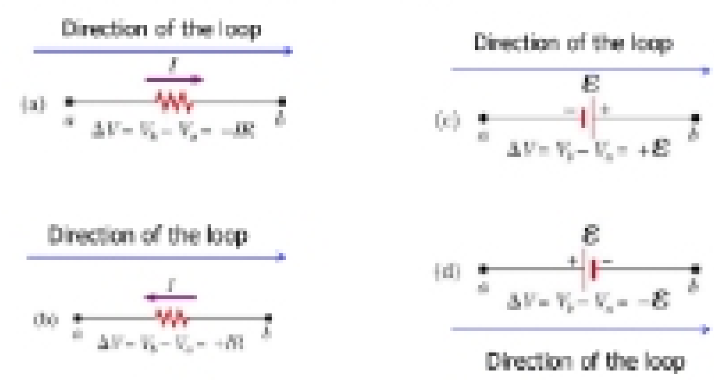
$$\Delta V_1 = V_c - V_b = -IR_1$$

$$\Delta V_2 = V_d - V_c = -IR_2$$

- Note the sign



More about the Loop Rule



Example: Loop Rule

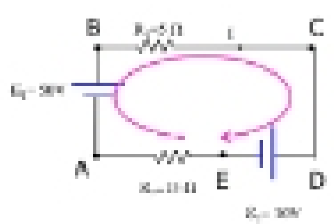
- Loop ABCDEA

$$\Delta V_{batt1} = V_B - V_A = \varepsilon_1 = 50 \text{ V}$$

$$\Delta V_1 = V_C - V_B = -IR_1$$

$$\Delta V_{batt2} = V_D - V_C = -\varepsilon_2 = -10 \text{ V}$$

$$\Delta V_2 = V_A - V_D = -IR_2$$



- Loop rule: $\Delta V_{batt1} + \Delta V_{batt2} + \Delta V_1 + \Delta V_2 = 0$

$$50 - IR_1 - 10 - IR_2 = 0 \quad \Rightarrow 40 - IR_1 - IR_2 = 0$$

$$\Rightarrow 40 = IR_1 + IR_2 \quad \Rightarrow I = \frac{40}{R_1 + R_2} = \frac{40}{20} = 2 \text{ A}$$

Example – Kirchhoff's Rules

(4) Loop rule: (a) Loop 1 (loop bcfeb)

$$\Delta V_{batt\_10V} = 10 \text{ V}$$

$$\Delta V_{6\Omega} = -I_1 \times 6$$

$$\Delta V_{batt\_14V} = 14 \text{ V}$$

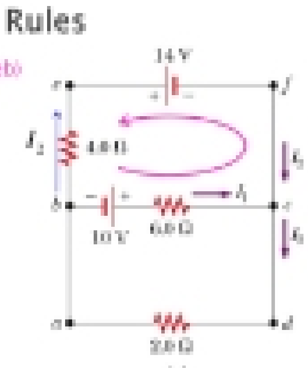
$$\Delta V_{4\Omega} = +I_2 \times 4$$

Loop rule: $\sum \Delta V = 0$

$$10 - 6I_1 + 14 + 4I_2 = 0$$

$$\Rightarrow 24 - 6I_1 + 4I_2 = 0 \quad \text{(devide both sides by 2)}$$

$$\Rightarrow 12 - 3I_1 + 2I_2 = 0$$



Example – Kirchhoff's Rules

b) Loop 2: loop bcdab

$$\Delta V_{batt\_10V} = 10 \text{ V}$$

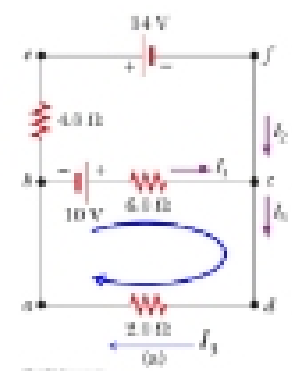
$$\Delta V_{6\Omega} = -I_1 \times 6$$

$$\Delta V_{2\Omega} = -I_3 \times 2$$

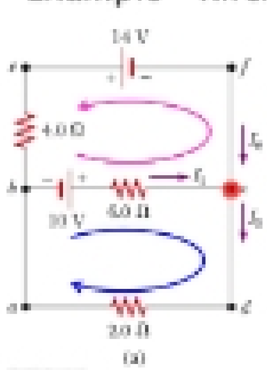
Loop rule: $\sum \Delta V = 0$

$$10 - 6I_1 - 2I_3 = 0$$

$$\Rightarrow 5 - 3I_1 - I_3 = 0$$



Example – Kirchhoff's Rules



Finally, we have

Junction rule: $I_1 + I_2 = I_3$ (1)

Loop rule 1): $12 - 3I_1 + 2I_2 = 0$ (2)

Loop rule 2): $5 - 3I_1 - I_3 = 0$ (3)

Three unknown, three equations:

$$\begin{cases} I_1 + I_2 = I_3 & (1) \\ 12 - 3I_1 + 2I_2 = 0 & (2) \\ 5 - 3I_1 - I_3 = 0 & (3) \end{cases}$$

Example – Kirchhoff's Rules

$$\begin{cases} I_1 + I_2 = I_3 & (1) \\ 12 - 3I_1 + 2I_2 = 0 & (2) \\ 5 - 3I_1 - I_3 = 0 & (3) \end{cases}$$

From eq (1): $I_3 = I_1 + I_2$

Eliminate I_3 from eq (3) using eq (1)

$$5 - 3I_1 - (I_1 + I_2) = 0$$

$$5 - 4I_1 - I_2 = 0$$

$$I_2 = 5 - 4I_1$$

Substitute the above into eq (2)

$$12 - 3I_1 + 2(5 - 4I_1) = 0$$

$$22 - 11I_1 = 0$$

$$I_1 = 2 \text{ A}$$

$$\Rightarrow I_2 = 5 - 4I_1 = -3 \text{ A}$$

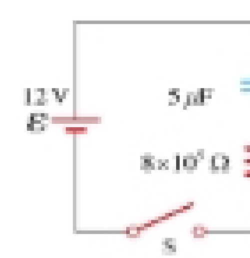
Finally,

$I_1 = 2$ A

$I_2 = -3$ A

$I_3 = I_1 + I_2 = -1$ A

Example



(1) Time constant of the circuit

$$\tau = RC = (8 \times 10^5) \times (5 \times 10^{-6}) = 4.0 s$$

(2) Maximum charge on the capacitor after the switch is closed

$$Q = \varepsilon C = 12 \times (5 \times 10^{-6}) = 60 \times 10^{-6} \text{ C} = 60\ \mu\text{C}$$

(3) Charge on the capacitor at t=6 sec (the switch is closed when t=0)

$$q(t) = Q(1 - e^{-t/\tau})$$

$$q(t = 6s) = Q(1 - e^{-6/4}) = (6 \times 10^{-5}) \times (1 - e^{-6.0/4.0}) = 4.7 \times 10^{-5} \text{ C}$$

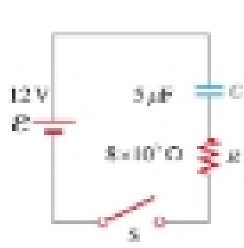
(4) Potential difference across the resistor at t=6 sec

a. potential drop across the capacitor at t=6 sec:

$$\Delta V_C = \frac{q}{C} = \frac{4.7 \times 10^{-5}}{5 \times 10^{-6}} \approx 9.3 \text{ V}$$

b. potential drop across the resistor

$$\varepsilon = \Delta V_R + \Delta V_C \quad \Rightarrow \Delta V_R = \varepsilon - \Delta V_C = 12 - 9.3 = 2.7 \text{ V}$$



(5) The current passing the resistor at t=6 sec: Ohm's law

$$I(t = 6s) = \frac{\Delta V_R}{R} = \frac{2.7}{8 \times 10^5} = 3.4 \times 10^{-6} \text{ A}$$

$\tau = 4 s$

$Q = 60\ \mu\text{C}$

$q(t = 6s) = 4.7 \times 10^{-5}$ C