ENERGY OF A SYSTEM

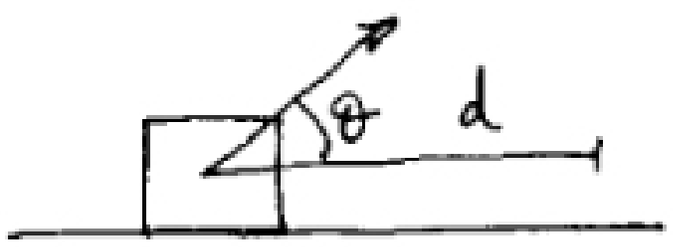
7.1. WORK DONE BY A CONSTANT FORCE

$$W = \vec{F} \cdot \vec{d} = F d \cos\theta$$

[constant force]

$$W = \int \vec{F}\, d\vec{r} = \vec{F} \int d\vec{r} = \vec{F}\vec{d}$$

[varying force : general case]



$$[W] = J = N\,m = kg\,\frac{m}{s^2}\cdot m = kg\,\frac{m^2}{s^2}$$

Note that work can be positive, negative and zero, even in the case we have displacement.

$0 \leq \theta \leq 90^\circ \quad W > 0$

$\theta = 90^\circ \quad W = 0$

$90^\circ \leq \theta \leq 180^\circ \quad W < 0$

<u>TOTAL WORK</u> :

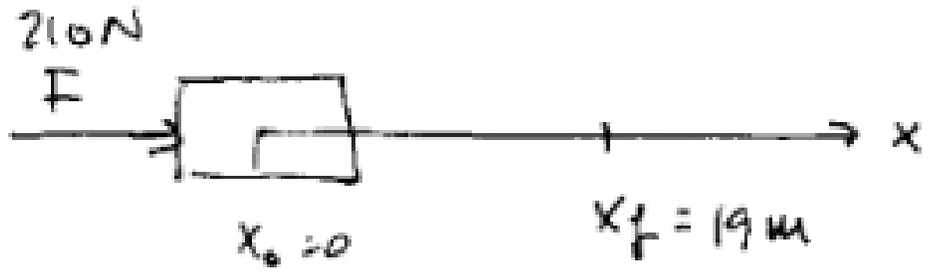
If there are several forces acting on an object that is displaced by $\vec{d}$, the total work is,

$$W_T = \sum_i W_i = \sum_i \vec{F}_i \vec{d} = \sum_i F_i d \cos\theta_i$$

Example

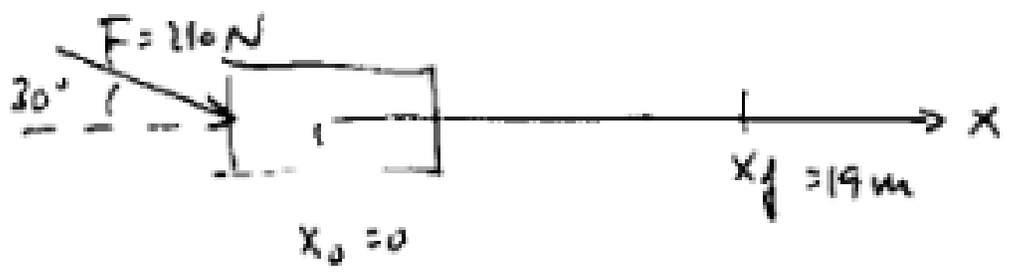
Work done by a $F = 210N$ force in moving an object 19 m when $\theta = 0$ and $\theta = 30°$

a)

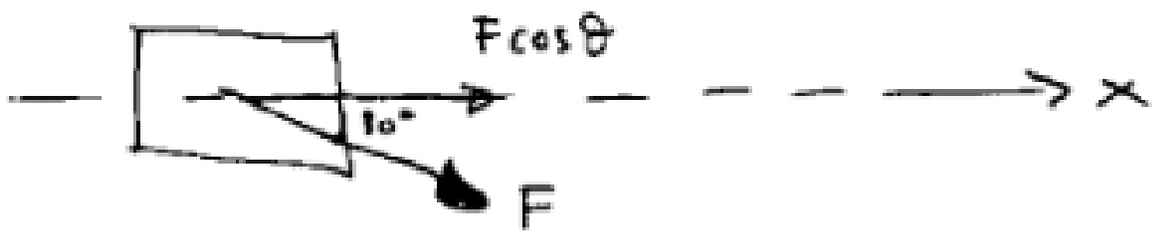


$$W = \vec{F} \cdot \vec{d} = F \cdot d \cos\theta = F \cdot d \cos 0° = Fd = 210\,N \cdot 19\,m = 4 \times 10^3$$

b)

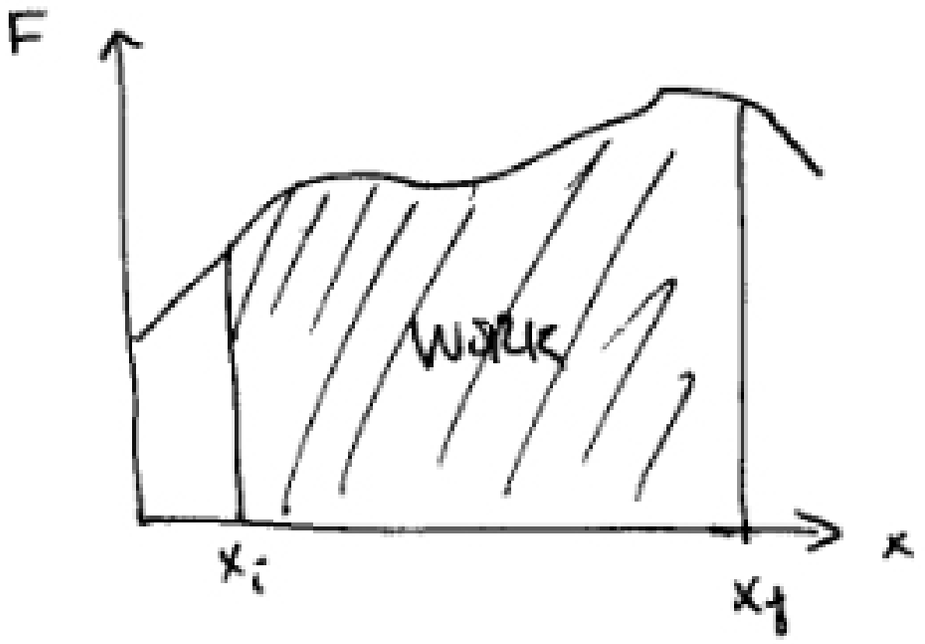


Only the projection of the force in the direction of movement will generate work.



$$W = (F \cos\theta)\, d = 210\,N \cdot \cos 30° \cdot 19\,m = \underline{3.5 \times 10^3\ J}$$

WORK DONE BY A VARYING FORCE



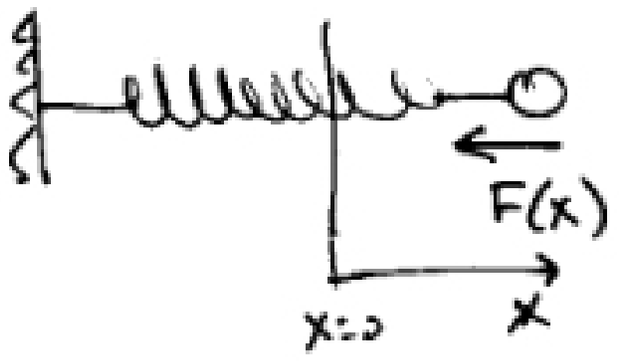
$$W = \int_{x_i}^{x_f} F\,dx$$

where $F(x)$

Total work:

$$W_N = \sum_i W_i = \int_{x_i}^{x_f} \left[\sum_i F_i(x)\right] dx$$

WORK DONE BY A SPRING



$$F_{sp} = -kx \quad = -F_{app}$$

$$W = \int_0^x (+kx)\,dx = \frac{1}{2}kx^2$$

F_{app}

$$W_{sp} = \frac{1}{2}kx^2$$