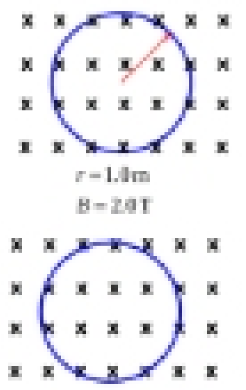
Example: Magnetic Flux



Magnetic flux: $\theta = 0°$; $\cos(0°) = 1$

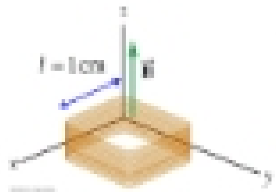
$$\Phi_B = BA\cos(\theta) = B\times(\pi r^2) = 2.0\times(3.14\times1^2) = 6.28\,\text{Wb}$$

If the angle between the magnetic field and the normal direction of the plane is 60°,

$$\Phi_B = BA\cos(\theta) = B\times(\pi r^2)\times\cos(60°) = 2.0\times(3.14\times1^2)\times0.5 = 3.14\,\text{Wb}$$

Example: Faraday's Law

- A coil in a changing magnetic field
  - Number of turns: $N = 25$
  - $l = 1\text{cm}$
  - B changes from 0 to 0.5 T in 1 second
  - Resistance of coil $r = 0.5\,\Omega$



(1) What is the emf induced during this one second?
(2) What is the magnitude of the current?
(3) What is the direction of the current?

$\Phi_B(initial) = 0$ $\quad A = (0.01)^2 m^2$

$\Phi_B(final) = BA = 0.5\times(0.01)^2 = 5\times10^{-5}\,\text{Wb}$

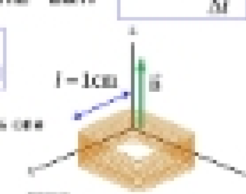
Example: Faraday and Lenz' Law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$N = 25$; $l = 1\text{cm}$; $r = 0.5\,\Omega$
B changes from 0 to 0.5 T in 1 second

(1) What is the emf induced during this one second?

$\Delta\Phi_B = \Phi_B(final) - \Phi_B(initial) = 5\times10^{-5}\,\text{Wb}$

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = -25\times\frac{5\times10^{-5}}{1} = -1.25\times10^{-3}\,\text{V}$$



$\theta = 0°$

The angle between the normal direction of the plane and the magnetic field

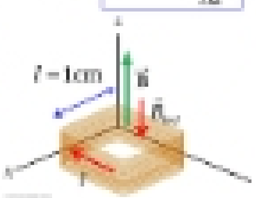
$$\begin{cases}\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}\\ \Phi_B = BA\cos(\theta)\end{cases}$$

Example: Faraday and Lenz' Law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$N = 25$; $l = 1\text{cm}$; $r = 0.5\,\Omega$
B changes from 0 to 0.5 T in 1 second

(2) What is the magnitude of the current?

$$I = \left|\frac{\varepsilon}{r}\right| = \frac{1.25\times10^{-3}}{0.5} = 2.5\times10^{-3}\,\text{A}$$



$\theta = 0°$

(3) Direction of the current
- induce a magnetic field opposite to the original field direction
- right hand rule

The angle between the normal direction of the plane and the magnetic field

Example: Motional emf

(1) Motional emf?

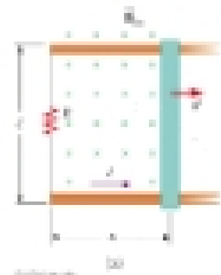
$$\varepsilon = Blv = 0.25\times0.5\times2 = 0.25\,\text{V}$$

(2) Current in the circuit?

$$I = \frac{\varepsilon}{R} = \frac{0.25}{0.5} = 0.5\,\text{A}$$

(3) Power delivered to the resistor?

$$P = I\varepsilon = 0.5\times0.25 = 0.125\,\text{W}$$



$l = 0.5\,\text{m}$
$v = 2\,\text{m/s}$
$B = 0.25\,\text{T}$
$R = 0.5\,\Omega$

Example: AC Generator $\varepsilon = \varepsilon_{max}\sin(\omega t)$, $\varepsilon_{max} = NBA\omega$

- An AC generator
  - 8-turn coil
  - Resistance of the circuit: 10 Ω
  - Area of the coil = 0.1 m²
  - B = 0.5 T
  - Rotation frequency: 60Hz

(1) What is maximum induced emf ε_{max}?
(2) What is maximum current I_{max}?
(3) What is the emf and current at t=0.01 s?
(4) What is the maximum torque that the external force must apply to keep coil rotating?

Example: Generator

$\varepsilon = \varepsilon_{max}\sin(\omega t)$
$\varepsilon_{max} = NBA\omega$
$N = 8$; $A = 0.1\,\text{m}^2$; $B = 0.5\,\text{T}$
$f = 60\,\text{Hz}$
$R = 10\,\Omega$

(1) What is maximum induced emf?

$$\varepsilon_{max} = NBA\omega \quad (\omega = 2\pi f)$$
$$= 8\times0.5\times0.1\times(2\pi f) = 0.4\times(6.28\times60) = 151\,\text{V}$$

(2) What is maximum current?

$$I_{max} = \frac{\varepsilon_{max}}{R} = \frac{151}{10} = 15.1\,\text{A}$$

Example: Generator

(3) What is emf at t=0.01 second?

$$\varepsilon = \varepsilon_{max}\sin(\omega t) = 151\times\sin(2\pi f t) = 151\times\sin(6.28\times60\times0.01) = 151\times(-0.59) = -88.2\,\text{V}$$

What is the current at t=0.01 second?

$$I = \frac{\varepsilon(t=0.01)}{R} = \frac{-88.2}{10} = -8.82\,\text{A}$$

$N = 8$; $A = 0.1\,\text{m}^2$; $B = 0.5\,\text{T}$;
$f = 60\,\text{Hz}$; $R = 10\,\Omega$
⇒
$\varepsilon_{max} = NBA\omega = 151$
$I_{max} = 15.1\,\text{A}$

Example: Generator

(4) What is the maximum external torque to keep on rotating the coil?

Analysis:
- A current loop in a magnetic field will experience a torque due to the magnetic field, $\tau = NIAB$
- This torque will tend to slow down the rotation.
- To keep on the rotation, an external torque must be applied to balance the magnetic torque

Solution: The magnetic torque:

$$\tau_{max} = NBI_{max}A = 8\times0.5\times15.1\times0.1 = 6.0\,\text{N}\cdot\text{m}$$

$N = 8$
$B = 0.5\,\text{T}$
$I_{max} = 15.1\,\text{A}$
$A = 0.1\,\text{m}^2$

Example $\quad L = \dfrac{\mu_0 N^2 A}{l} \qquad \varepsilon = -L\dfrac{\Delta I}{\Delta t}$

- Consider a solenoid contain 300 turns. The length of the solenoid is 25 cm and its cross-sectional area is 4×10^{-4} m².

(1) What is the inductance of the solenoid?
(2) What is the self-induced emf if the current in the solenoid decreases from 50 A to 25 A in 0.5 seconds?

Solution: $L = \dfrac{\mu_0 N^2 A}{l} = 4\pi\times10^{-7}\times\dfrac{300^2\times4\times10^{-4}}{25\times10^{-2}} = 0.181\,\text{mH}$

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -0.181\times10^{-3}\times\frac{(25-50)}{0.5} = 9.05\,\text{mV}$$

Example: RL Circuit

- What is the time constant?

$$\tau = \frac{L}{R} = \frac{30\times10^{-3}}{0.15} = 0.2\,\text{s}$$

- What is the maximum current in the circuit?

$$I_{max} = \frac{\varepsilon}{R} = \frac{1.5}{0.15} = 10.0\,\text{A}$$

- What is the current at $t = \tau$ after the switch is closed?

$$I(t=\tau) = I_{max}(1-e^{-\tau/\tau}) = I_{max}(1-e^{-1}) = 10.0\times0.632 = 6.32\,\text{A}$$

Example: RL Circuit

- What are the voltages across the resistor and the inductor at t=0 and t=τ?

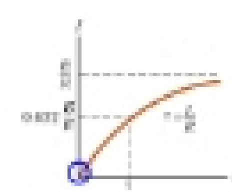
$\varepsilon = 0$, $I = 0 \Rightarrow \Delta V_R = IR = 0$

$\Delta V_L = \varepsilon - \Delta V_R = 1.5\,\text{V}$

- What are the voltages across the resistor and inductor at t=τ.

$\Delta V_R = IR \quad \leftarrow I(t=\tau) = 6.32\,\text{A}$
$= 6.32\times0.15$
$= 0.95\,\text{V}$

$\Delta V_L = \varepsilon - \Delta V_R = 1.5 - 0.95 = 0.55\,\text{V}$



Faraday's Law

- If a circuit contains N tightly wound loops and the flux changes by $\Delta\Phi_B$ during a time interval Δt, then the emf induced is given by:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

- This is known as Faraday's law

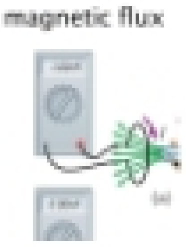
20.2 Faraday's Law and Electromag Induction (Demo)

- The induced emf = rate of change of magnetic flux

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

where $\Delta\Phi_B = \Phi_B(t+\Delta t) - \Phi_B(t)$

is the change in magnetic flux during the time interval Δt.



AC Generators

$\Phi_B = BA\cos(\omega t)$

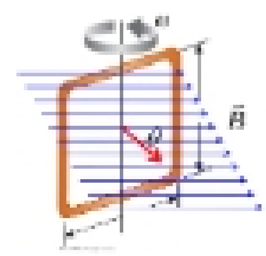
$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}$$

- It can be shown that

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = BA\omega\sin(\omega t)$$

FYI:
$\lim_{\Delta t\to0}\dfrac{\Delta\cos(\omega t)}{\Delta t} = \dfrac{d\cos(\omega t)}{dt} = -\omega\sin(\omega t)$

Calculus!



Red arrow: normal direction of the plane

AC Generators $\varepsilon = BA\omega\sin(\omega t)$

$\Phi_B = BA\cos(\omega t)$
$\varepsilon = BA\omega\sin(\omega t) = \varepsilon_{max}\sin(\omega t)$
$\varepsilon_{max} = BA\omega$

- For N-turn loop

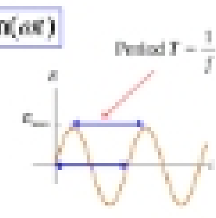
$\varepsilon = \varepsilon_{max}\sin(\omega t)$
$\varepsilon_{max} = NBA\omega$

- How to achieve a greater emf? $\quad \varepsilon_{max} = NBA\omega$
  - Stronger magnetic field: B
  - Larger loop: A
  - Faster rotation: ω
  - More turns (loops): N

AC Generators $\varepsilon = NBA\omega\sin(\omega t)$

$\varepsilon = \varepsilon_{max}\sin(\omega t)$; $\quad \varepsilon_{max} = NBA\omega$

N: number of turns
B: magnetic field
A: area of the loop
ω: angular speed of the rotation



- Plot of ε as a function of time:
  - sinusoidal function, alternates between positive and negative values → AC
- Amplitude: $\varepsilon_{max} = NBA\omega$
- frequency: $f = \dfrac{\omega}{2\pi}$
- period: $T = \dfrac{1}{f}$

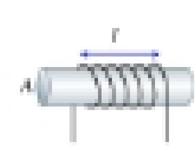
Self Inductance

- For a N-turn coil, if a current I results in a magnetic flux Φ_B in the coil, the inductance of the coil can be calculated by

$$L = N\frac{\Phi_B}{I}$$

- Inductance of a solenoid:

$$L = \frac{\mu_0 N^2 A}{l}$$



N: number of turns
l: length of the solenoid
A: area of the solenoid
$\mu_0 = 4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}$: permeability of free space

RL Circuit

$$I = \frac{\varepsilon}{R}(1-e^{-t/\tau})$$

- The time constant, τ, is the time required for the current in the circuit to reach 63.2% of its final value

$$\tau = \frac{L}{R}$$

when $t = \tau$, $(1-e^{-t/\tau}) = (1-e^{-1}) = 0.632$

- If you wait long enough, the current will reach a maximum value

$$I(t\to\infty) = I_{max} = \frac{\varepsilon}{R}$$