<u>AMS 572 Lecture Notes #3</u>

<u>Sept. 17, 2010.</u>

<u>One-sample Inference on Population Mean</u>

④ When the population is normal, and the population variance <u>known</u>

Data : $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$

<iii> **Hypothesis test** $\begin{cases} H_0: \mu = \mu_0 \\ H_a: \mu > \mu_0 \end{cases} \Leftrightarrow \begin{cases} H_0: \mu \le \mu_0 \\ H_a: \mu > \mu_0 \end{cases}$

Example:

$H_0: \mu \le 5'7''$ (null hypothesis)

$H_a: \mu > 5'7''$ (alternative hypothesis) : This is usually your hypothesis if you are conducting the test.

Very similar to a law suit:

e.g) The famous O.J. Simpson trial

H_0 : OJ is innocent

H_a : OJ is guilty

		The truth	
		H_0 : OJ innocent	H_a OJ guilty
Jury's Decision	H_0	Right decision	Type II error
	H_a	Type I error	Right decision

The significance level and three types of hypotheses.

P(Type I error) = α ← significance level of a test (*Type I error rate)

1. $H_0: \mu = \mu_0$ ⇔ $H_0: \mu \le \mu_0$

   $H_a: \mu > \mu_0$ $\quad$ $H_a: \mu > \mu_0$

2. $H_0: \mu = \mu_0$ ⇔ $H_0: \mu \ge \mu_0$

   $H_a: \mu < \mu_0$ $\quad$ $H_a: \mu < \mu_0$

3. $H_0: \mu = \mu_0$

   $H_a: \mu \ne \mu_0$

Now we derive the hypothesis test for the first pair of hypotheses.

$H_0: \mu = \mu_0$

$H_a: \mu > \mu_0$

Data : $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2), \sigma^2$ is known and the given a significance level α (say, 0.05). Let's derive the test. (That is, derive the decision rule)

Two approaches (*equivalent):

- Likelihood Ratio Test
- Pivotal Quantity Method

***Now we will first demonstrate the **Pivotal Quantity** Method.

1. We have already derived the PQ when we derived the C.I. for μ

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$ is our P.Q.

2. The test statistic is the PQ with the value of the parameter of interest under the null hypothesis (H_0) inserted:

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \overset{H_0}{\sim} N(0,1)$$ is our test statistic.

That is, given $H_0: \mu = \mu_0$ in true $\Rightarrow Z_0 = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$

3. * Derive the decision threshold for your test based on the Type I error rate the significance level α

For the pair of hypotheses:

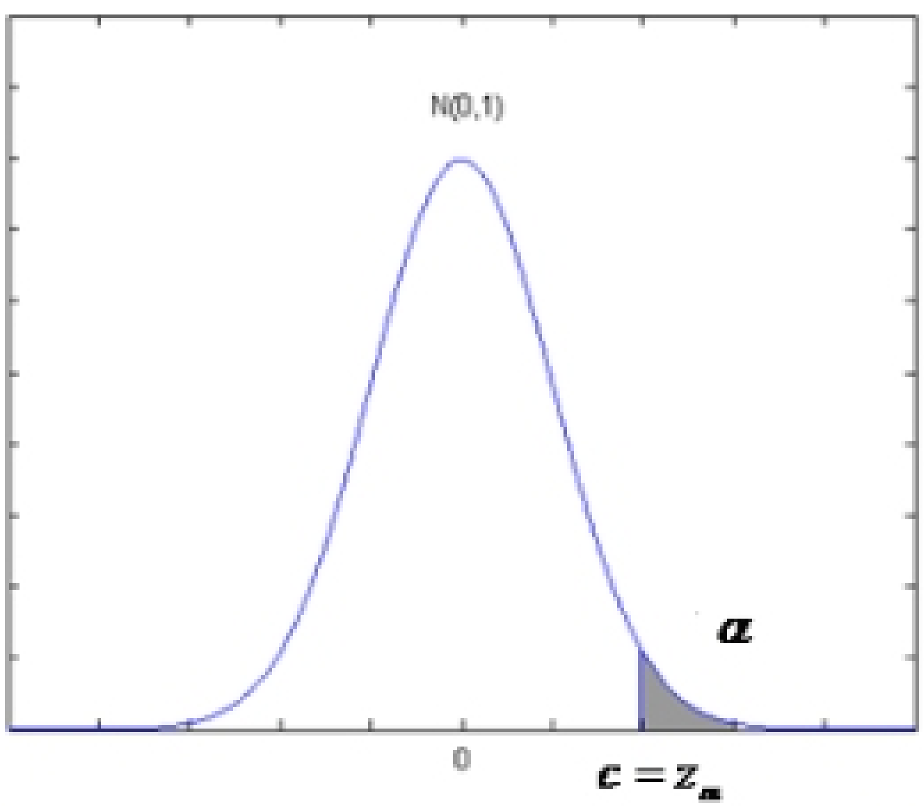
$$H_0: \mu = \mu_0 \qquad \textbf{versus} \qquad H_a: \mu > \mu_0$$

It is intuitive that one should reject the null hypothesis, in support of the alternative hypothesis, when the sample mean is larger than μ_0. Equivalently, this means when the test statistic Z_0 is larger than certain positive value c - the question is what is the exact value of c -- and that can be determined based on the significance level α—that is, how much Type I error we would allow ourselves to commit.

Setting:

P(Type I error) = P(reject $H_0 | H_0$) = $P(Z_0 \geq c \mid H_0: \mu = \mu_0) = \alpha$

We will see immediately that $c = z_\alpha$ from the pdf plot below.



∴ At the significance level α, we will reject H_0 in favor of H_a if $Z_0 \geq Z_\alpha$

Other Hypotheses