Physics 235
Summary Sheet #8: DC Circuits

Current, drift velocity, current density

1. A **current** is any motion of charge from one region to another.

2. The electrons within a wire (or whatever) are generally stationary; their random motion leads them to bang into surrounding molecules and therefore not move very much. However, if an electric field is applied across the wire in one direction, their net movement will carry them in the direction of the field at some slow rate, v_d, which is called the **drift velocity.**

3. Since the charge carriers are electrons, which are negatively-charged, and convention has decided that current is carried by positive charge carriers, the direction of the **conventional current** is actually opposite from the direction of motion of the electrons.

4. The standard unit of current is the **Ampere (A)** which is defined as one coulomb of charge [passing through an area] per second.

5. The **current** in a wire is related to the drift velocity of the charge carriers by the following expression, where *q* is the charge of an individual charge carrier, *n* is their number density, and *A* is the cross-sectional area of the wire.

$$I = \frac{dQ}{dt} = nqv_d A\,dt$$

6. The current per cross-sectional area is called the **current density.**

$$J = \frac{I}{A} = nqv_d$$

In vector form this is:

$$\vec{J} = nq\vec{v}_d$$

Resistivity, resistance, and Ohm's law

1. The dependence of current density on the electric field and the properties of the material has the potential to be very complex. In our analysis, we will focus on materials where the current density is basically linearly proportional to the electric field. These materials adhere to a relationship known as **Ohm's law.** In the expression below, ρ is a quantity called the **resistivity.**

$$\rho = \frac{E}{J}$$

A perfect conductor would have zero resistivity, and a perfect insulator would have infinite resistivity. The reciprocal of resistivity is **conductivity**. The units of resistivity are **Ohm-meters (Ω m)**. One Ohm is defined to be one Volt per Ampere. (1 Ω = 1 V/A)

2. Resistivity is dependent on temperature. For a *metallic conductor*, the resistivity nearly always *increases* with increasing temperature. For *nonmetals*, the resistivity generally *decreases* with increasing temperature. For some odd materials, called *superconductors*, the resistivity decreases smoothly as they are cooled (as though they were metals) and then suddenly drops to zero at a particular cutoff temperature.

3. The value of the current, I, is related to the potential difference between the ends of the conductor by a factor called the **resistance (R)**. The total current in the conductor is given by $I = JA$, and the potential difference between the ends is $V = EL$ (if the electric field and current density are uniform throughout). Using the relationship above for resistivity, we see that:

$$\frac{V}{I} = \frac{\rho L}{A} = R$$

where R is the resistance. Its units are Ohms. (You should check this for yourself.)

4. A circuit device made to have a specific value of resistance is called a **resistor.**

Electromotive force (emf)

1. For a conductor to have a steady current, it must be part of a closed path called a **complete circuit.** Since the current will flow in a direction of decreasing potential energy if left to its own devices, there must be something in that closed path that *increases* the potential energy so the current will constantly flow. Otherwise, the charges would just settle to their minimum energy state and stay there.

2. Devices that serve to increase the charge carriers' potential energy are called sources of **electromotive force,** or **emf.** In a device such as a battery, generator, or solar cell, charge actually travels "uphill". It does this by converting another source of energy (chemical energy, light energy, etc.) into electrical potential energy, which it then feeds to the charge carriers in the circuit.

3. An ideal source of emf will convert all of its energy to electrical potential energy with energy loss to dissipation within its own "parts".

$$V_{ab} = \xi$$

However, real sources of emf also have some **internal resistance**, so the potential difference between their terminals is not quite equal to their intrinsic electromotive force.

$$V_{ab} = \xi - I r$$

4. The net change in potential around a complete circuit must be zero, for the same reason that you can't walk around in a circle and end up a foot higher off the ground than you started.

Energy and power in circuits

1. Let's say you have a circuit element (resistor, battery, etc.) that has some potential difference across its terminals. As charge passes through that element, the internal electric field of the element does work on the charge. It either increases the potential energy of that charge (battery) or decreases it (resistor). Alternatively, we can think of this as energy being transferred to a circuit element (resistor) or taken from it (battery). The *rate at which energy is delivered to or extracted from a circuit element* is called the **power (P)**, which has units of **watts (J/s)**:

$$P = V_{ab} I$$

2. Since the potential difference across the terminals of a resistor is *IR*, we can calculate the **power dissipated in a resistor.**

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$$

   In a resistor, this energy is converted into heat (the thermal motions of the atoms in the resistor material).

Resistors in series and parallel

1. Lots of things that run on electricity, such as light bulbs, household appliances, and the like, are basically just resistors.

2. For resistors in **series**, the current is the same in each one. The equivalent resistance of any number of resistors in series equals the sum of their individual resistances.

$$R_{tot} = R_1 + R_2 + R_3 + \cdots$$

3. For resistors in **parallel**, the potential difference is the same for each one. The reciprocal of the equivalent resistance of any number of resistors in parallel equals the sum of the reciprocals of their individual resistances.

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Kirchhoffs rules

1. A **junction** in a circuit is a point where three or more conductors meet.

2. **Kirchhoffs junction rule** states that the algebraic sum of the currents into any junction is zero.

3. A **loop** is any closed conducting path.

4. **Kirchhoffs loop rule** states that the algebraic sum of the potential differences in any loop, including those associated with emfs and those of resistive elements, must equal *zero*. This rule is based on the simple principle of conservation of electric charge.