Homework 25 – Power Series

1) Show that the power series a) – c) have the same radius of convergence. Then show that (a) diverges at both endpoints, (b) converges at one endpoint but diverges at the other, and (c) converges at both endpoints.

a) $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

b) $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

c) $\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$

2) Find the radius and interval of convergence.

a) $\sum_{n=0}^{\infty} nx^n$

b) $\sum_{n=0}^{\infty} n(x-3)^n$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{n!}$

d) $\sum_{n=0}^{\infty} \frac{(n!)x^n}{5^n}$

e) $\sum_{n=15}^{\infty} \frac{x^{2n+1}}{3n+1}$

f) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{2^n n}$

g) $\sum_{n=1}^{\infty} (-1)^n n^5 (x-7)^n$

h) $\sum_{n=12}^{\infty} e^n (x-2)^n$

3) Use $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x|<1$ to expand the function in a power series with center $c=0$ and determine the interval of convergence.

a) $f(x) = \frac{1}{5-x}$

b) $f(x) = \frac{1}{16+2x^3}$

4)

a) Use differentiation to show that $(1-x)^{-2} = \sum_{n=1}^{\infty} nx^{n-1}$ for $|x|<1$.

b) Use the result in a) to evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

5)

a) Use the power series for $y = e^x$ to show that

$$\frac{1}{e} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots$$

b) Use your knowledge of alternating series to find an N such that the partial sum S_N approximates e^{-1} to within an error of at most 10^{-3}. You may confirm your answer with a calculator by computing both S_N and e^{-1}.