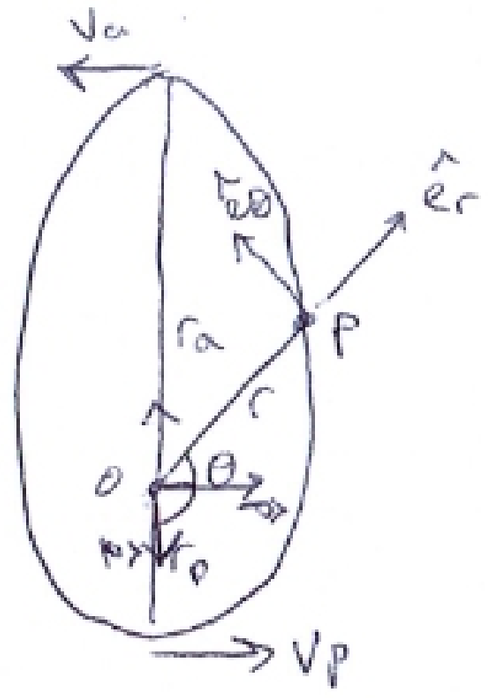
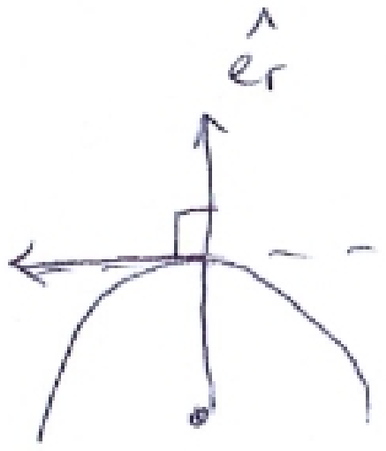
3.



$\bar{v} = v_r \hat{e}_r + v_\theta \hat{e}_\theta$

$v_r = v \sin\gamma$

$v_{r,p} =$



a) Work done from r_a to r_p

$$W = \int_{t_1}^{t_2} \bar{F} \cdot \bar{v}^{ep} \, dt$$

FBD:

$$F_g = -\frac{\mu m}{r^2} \hat{e}_r$$

Kinematics:

$$\bar{R}^{op} = r \hat{e}_r$$

$$\bar{v}^{ep} = \dot{r} \hat{e}_r + \dot{\theta} \hat{e}_n \times r \hat{e}_r = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$$

$$W = \int_{t_1}^{t_2} \left(-\frac{\mu m}{r^2} \hat{e}_r\right) \cdot \left(\dot{r} \hat{e}_r + r \cancel{\dot{\theta}}^{\,0} \hat{e}_\theta\right) dt$$

$$= \int_{t_1}^{t_2} -\frac{\mu m}{r^2} \dot{r} \, dt \qquad \dot{r} = \frac{dr}{dt} \qquad \dot{r} dt = dr$$

$$= -\mu m \int_{t_1}^{t_2} \frac{dr}{r^2} = -\mu m \left(-\frac{1}{r}\right)_{t_1}^{t_2} = \boxed{\mu m \left(\frac{1}{r_p} - \frac{1}{r_a}\right) = W}$$

3. b) $\boxed{W = \Delta T = T(t_2) - T(t_1)}$

c) $W = \mu m \left(\frac{1}{r_p} - \frac{1}{r_a} \right)$

$T = \frac{1}{2} m \, \underline{V}^{op} \cdot \underline{V}^{op} \qquad \underline{V}^{op} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$

$T = \frac{1}{2} m \left(\dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta \right) \cdot \left(\dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta \right)$

$T = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right)$

$T(t_1) = \frac{1}{2} m r_a^2 \dot{\theta}_a^2 \qquad \dot{\theta} = \frac{v \cos\gamma}{r} = \frac{V}{r} \qquad \dot{\theta}_a = \frac{V_a}{r_a}$

$\dot{\theta}_p = \frac{V_p}{r_p}$

$T(t_1) = \frac{1}{2} m V_a^2$

$T(t_2) = \frac{1}{2} m V_p^2$

$\Delta T = T(t_2) - T(t_1) = \frac{1}{2} m V_p^2 - \frac{1}{2} m V_a^2$

$= \frac{1}{2} m \left(V_p^2 - V_a^2 \right)$

$\boxed{\mu m \left(\frac{1}{r_p} - \frac{1}{r_a} \right) = \frac{1}{2} m \left(V_p^2 - V_a^2 \right)}$

3.d) $\mu\left(\frac{1}{r_p} - \frac{1}{r_a}\right) = \frac{1}{2}\left(v_p^2 - v_a^2\right)$

$$\frac{1}{2}v_p^2 = \mu\left(\frac{1}{r_p} - \frac{1}{r_a}\right) + \frac{1}{2}v_a^2$$

$$\boxed{v_p = \sqrt{2\mu\left(\frac{1}{r_p} - \frac{1}{r_a}\right) + v_a^2}}$$

e) $E = T + V$

$$T_1 + V_1 = T_2 + V_2$$

$$V_1 = -\frac{\mu m}{r_a} \qquad V_2 = -\frac{\mu m}{r_p}$$

$$\frac{1}{2}mv_a^2 - \frac{\mu m}{r_a} = \frac{1}{2}mv_p - \frac{\mu m}{r_p}$$

$$\frac{1}{2}v_p^2 = \frac{1}{2}v_a^2 + \frac{\mu m}{r_p} - \frac{\mu}{r_a} \qquad \frac{\mu}{r_p}$$

$$\boxed{v_p = \sqrt{2\mu\left(\frac{1}{r_p} - \frac{1}{r_a}\right) + v_a^2}}$$