Name ________________________ **2270-1**

Introduction to Linear Algebra 2270-1
Sample Midterm Exam 2 Fall 2007
Exam Date: 31 October

Instructions. This exam is designed for 50 minutes. Calculators, books, notes and computers are not allowed.

1. (Matrices, determinants and independence) Do two parts.

(a) Prove that the pivot columns of A form a basis for $\mathbf{im}(A)$.

(b) Suppose A and B are both $n \times m$ of rank m and $\mathbf{rref}(A) = \mathbf{rref}(B)$. Prove or give a counterexample: the column spaces of A and B are identical.

Start your solution on this page. Please staple together any additional pages for this problem.

2. (Kernel and similarity) Do two parts.

(a) Illustrate the relation $\mathbf{rref}(A) = E_k \cdots E_2 E_1 A$ by a frame sequence and explicit elementary matrices for the matrix

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{pmatrix}.$$

(b) Prove or disprove: $\mathbf{ker}(\mathbf{rref}(BA)) = \mathbf{ker}(A)$, for all invertible matrices B.

Start your solution on this page. Please staple together any additional pages for this problem.

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3. (Independence and bases) Do two parts.

(a) Let A be a 12×15 matrix. Suppose that, for any possible independent set $\mathbf{v}_1, \ldots, \mathbf{v}_k$, the set $A\mathbf{v}_1, \ldots, A\mathbf{v}_k$ is independent. Prove or give a counterexample: $\mathbf{ker}(A) = \{\mathbf{0}\}$.

(b) Let V be the vector space of all polynomials $c_0 + c_1x + c_2x^2$ under function addition and scalar multiplication. Prove that $1 - x$, $2x$, $(x-1)^2$ form a basis of V.

Start your solution on this page. Please staple together any additional pages for this problem.