Math 231E. Fall 2013. Midterm 2 Practice Problems.

Problem 1. Compute the derivatives of the following functions

a. $f(x) = \cos(x^2)e^{\sin(x)}$

b. $f(x) = \arctan(x)$

c. $f(x) = \ln(e^x(1 + x^2 + x^3))$

Problem 2. Consider the polynomial $P(x) = 8x^3 - 36x^2 + 46x - 15$. We want to find the roots of this polynomial. Find "brackets" for all of the roots of this polynomial of the form $[k, k+1]$, i.e. find three integers k_1, k_2, k_3 such that you can show there is a root r with $k_1 < r < k_1 + 1$. Use any calculus facts we have covered so far in class.

Problem 3. Evaluate the following limits, or show they do not exist:

a. $\displaystyle\lim_{x\to 1} \frac{1 - x^2 + 4x^3}{\sin(x)}$

b. $\displaystyle\lim_{x\to 0} \frac{|x|}{\sin(x)}$

c. $\displaystyle\lim_{x\to 0} \cos\left(\frac{e^{x^3} - 1}{x - \sin(x)}\right)$

d. $\displaystyle\lim_{x\to 3} \int_3^x e^{-t^2}\,dt.$

e. $\displaystyle\lim_{x\to 3} \frac{1}{x-3}\int_3^x e^{-t^2}\,dt.$

f. $\displaystyle\lim_{x\to 0} \exp(x/\sin(x))$

Problem 4. Let us define

$f(x) =$ "the second digit after the decimal point in the decimal expansion for x".

Does $\displaystyle\lim_{x\to 0} f(x)$ exist? If so, what is it? If not, why not?

Problem 5. A parallelogram has sides of length one meter that are hinged at the ends. At the moment when the height is $\frac{\sqrt{2}}{2}$ m it is decreasing at 1 m/s. How fast is the angle changing?

Problem 6. Evaluate the following definite integrals

a. $\displaystyle\int_0^\pi x\cos(x)\,dx$

b. $\displaystyle\int_1^2 x^2 \ln x\,dx$

c. $\displaystyle\int_{-1}^0 x(1+x)^{2012}\,dx$

Problem 7. Evaluate the following indefinite integrals

a. $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$

b. $\displaystyle\int \sqrt{x}e^{\sqrt{x}}\,dx$

c. $\displaystyle\int \frac{\sin(\theta)}{1-\cos^2(\theta)}\,d\theta$

Problem 8. A particle at the end of a string spins in a circle about the origin. At the instant that the particle is at the point $(1/2\,\text{m}, \sqrt{3}/2\,\text{m})$, the x coordinate is decreasing at a rate of $1\,\text{m/s}$.

a. Find the rate of change of the y coordinate.

b. Find the rate of change of the angle θ.

Problem 9. Some questions about continuity:

a. If $f(x)$ is continuous, is it necessarily true that $(f(x))^2$ is continuous?

b. If $f(x)$ is not continuous is it necessarily true that $(f(x))^2$ is not continuous?

Problem 10. Consider the function $f(x) = x^{2013}+x+1$. Show that there is a point $c \in (0,1)$ such that $f'(c) = 2013$.

Problem 11. Find the first three non-zero terms in the Taylor series for $f(x) = \arctan(x)$ about the point $a = 0$.

Problem 12. Compute the upper Riemann sum U_4 for the function $f(x) = x^3 + x^2$ for the interval $[1,2]$ with $n = 4$ subintervals.

Problem 13. The largest box the United States Postal Service (USPS) will accept is one whose combined length (longest dimension) and girth (circumference around the box perpendicular to the length) is 108 inches. What is the largest possible box volume, in cubic feet, that the USPS will accept?

Problem 14. A metal drum in the shape of a right circular cylinder **with no top** must be built to have surface area $3\pi\,\text{ft}^2$. What height h and base radius r will maximize the volume of the cylinder ?