Math 140
Exam I Study Guide

The Limit of a Function

"The limit of (x) approaches a, equals L" means that, as the value of x gets closer to a (but not equal to a from either side of a) the value of f(x) gets closer and closer to L

$\lim_{x \to a} f(x) = L$ if and only if:

$\lim_{x \to a+} f(x) = L$ "the limit as x approaches a from the right"

$\lim_{x \to a-} f(x) = L$ "the limit as x approaches a from the left"

non-zero/zero – infinite limit

Squeeze Theorem
If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a), and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$

Continuity
If $\lim_{x \to a} f(x) = f(a)$, then f is continuous at a

1. F(a) is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

These functions are continuous at every number in their domain:
- Polynomials
- Rational functions
- Root functions
- Trigonometric functions

If f and g are continuous at a, and c is a constant, the these are also continuous at a:

F+g f-g fg f/g if g(a) is not 0

If g is continuous at a, and f is continuous at g(a), then (f*g)(x) – f(g(x)) is continuous at a

If f(x) is not continuous at a, then f has a discontinuity at a.
Types of Discontinuities:
1. Removable – can be removed by redefining f at one point. Usually when a 0 in the denominator divides out with something in the numerator
   F(x) = (x-4)(x+7)/(x-4) Removable discontinuity at x=4
2. Jump – piecewise, absolute value, etc. Right and left limits are not equal.
3. Infinite – function goes to positive or negative infinity

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Intermediate Value Theorem

Suppose that f is continuous on the closed interval [a,b] and that f(a) is not equal to f(b). Let N be any number between f(a) and f(b), then there exists a number c in the open interval (a,b) such that f(c) = N

> Prove that a function has a root. If f is continuous on [a,b], and f(a)>0 and f(b)<0 (or vice versa), then intermediate value theorem tells us that there is some c in (a,b) such that f(c) =0.

Derivatives and Rates of Change

The slope of the tangent line to y=f(x) at the point (a,f(a)) is

$m = \lim_{x \to a} f(x) - f(a)/x-a$ $m = \lim_{h \to 0} f(a+h) - f(a)/h$

Derivative = rate of change = slope
Increasing function = positive derivative
Decreasing function = negative derivative

When is a function not differentiable?

1. If f is not continuous at a, then f is not differentiable at a
2. Limit from the left and right are different
3. At a cusp
4. Vertical tangent

If f(x)=c, then f'(x) = 0

Power rule: $x^n = nx^{n-1}$

Sum rule: f(x) + g(x) = f'(x) + g'(x)

Product rule: f(x)g(x) = f(x)g'(x) + g(x)f'(x)

Quotient rule: $f/g = gf' - fg'/g^2$ "Low D. High – High D. Low, Square the Bottom and off you go." (bottom*Derivative of the top – top*Derivative of the bottom. Square the bottom)

Trig Derivatives

Sin(x)	Cos(x)	Cos(x)	-Sin(x)
Tan(x)	$Sec^2(x)$	Csc(x)	-csc(x)cot(x)
Sec(x)	Sec(x)tan(x)	Cot(x)	$-csc^2(x)$

Chain rule: if h(x) = g(f(x)), then h'(x) = g'(f(x)) * f'(x), if g'(f(x)) and f'(x) exist

"the derivative of the outside time the derivative of the inside"

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Implicit Differentiation:
Derivative of y is y'. Differentiate the equation and solve for y'

Related Rates:
1. Read the question
2. Draw a picture (or several)
3. Identify variables and constants, and any other information you are given
4. Figure out what you want to find, and when
5. Find an equation that relates the variable whose derivative you want to find to other variables
6. Differentiate the equation with respect to t (time)
7. Substitute in known quantities and solve
8. Check for reasonableness