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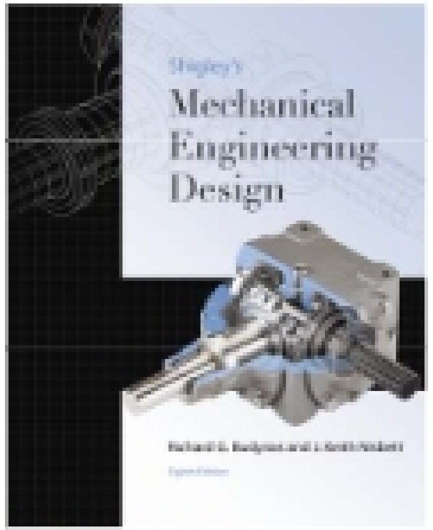
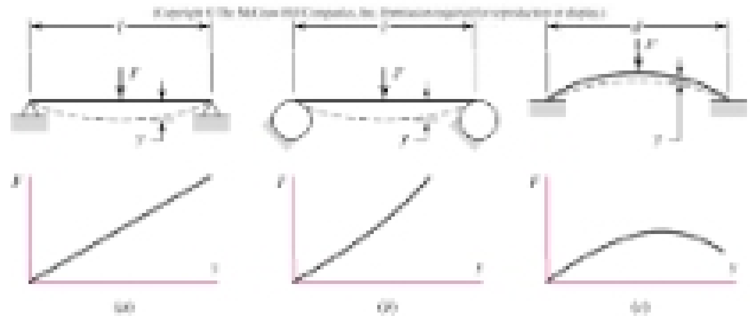
Chapter 4 Image Slides



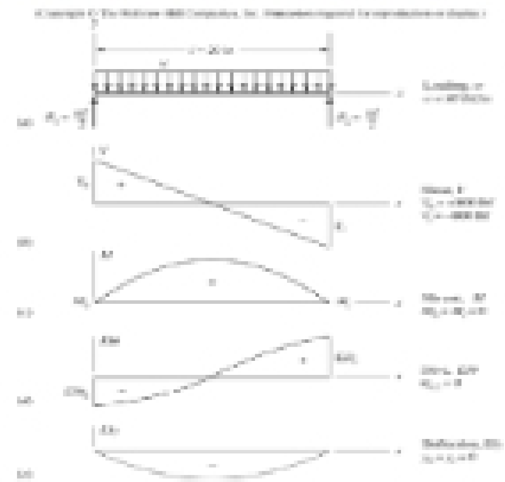
4-1 Springs



4-3 Deflection Due to Bending Equations

$$q/EI = y''''$$
$$V/EI = y'''$$
$$M/EI = y''$$
$$\theta = y'$$
$$y = f(x)$$

4-2 Deflection Due to Bending



EXAMPLE 4–1 For the beam in Fig. 4-2, the bending moment equation, for $0 \le x \le l$, is

$$M = \frac{wl}{2}x - \frac{w}{2}x^2$$

Using Eq. (4-12), determine the equations for the slope and deflection of the beam, the slopes at the ends, and the maximum deflection.

Solution Integrating Eq. (4-12) as an indefinite integral we have

$$EI\frac{dy}{dx} = \int M\,dx = \frac{wl}{4}x^2 - \frac{w}{6}x^3 + C_1 \quad (1)$$

where C_1 is a constant of integration that is evaluated from geometric boundary conditions. We could impose that the slope is zero at the midspan of the beam, since the beam and loading are symmetric relative to the midspan. However, we will use the given boundary conditions of the problem and verify that the slope is zero at the midspan. Integrating Eq. (1) gives

$$EIy = \iint M\,dx = \frac{wl}{12}x^3 - \frac{w}{24}x^4 + C_1x + C_2 \quad (2)$$

The boundary conditions for the simply supported beam are $y = 0$ at $x = 0$ and l. Applying the first condition, $y = 0$ at $x = 0$, to Eq. (2) results in $C_2 = 0$. Applying the second condition to Eq. (2) with $C_2 = 0$,

$$EIy(l) = \frac{wl}{12}l^3 - \frac{w}{24}l^4 + C_1 l = 0$$

Solving for C_1 yields $C_1 = -wl^3/24$. Substituting the constants back into Eqs. (1) and (2) and solving for the deflection and slope results in

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \quad (3)$$

$$\theta = \frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3) \quad (4)$$

Comparing Eq. (3) with that given in Table A-9, beam 7, we see complete agreement.

For the slope at the left end, substituting $x = 0$ into Eq. (4) yields

$$\theta|_{x=0} = -\frac{wl^3}{24EI}$$

and at $x = l$,

$$\theta|_{x=l} = \frac{wl^3}{24EI}$$

At the midspan, substituting $x = l/2$ gives $dy/dx = 0$, as earlier suspected.

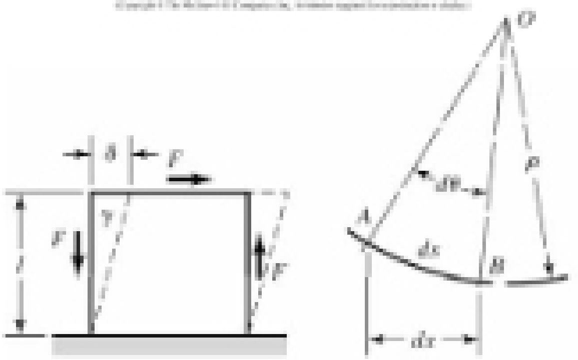
The maximum deflection occurs where $dy/dx = 0$. Substituting $x = l/2$ into Eq. (3) yields

$$y_{max} = -\frac{5wl^4}{384EI}$$

which again agrees with Table A-9-7.

Do Problem 4-12 on Board

4-8 Strain Energy Elements



(a) Pure shear element (b) Beam bending element

Table 4–1

Strain-Energy Correction Factors for Shear

Source: Richard G. Budynas, *Advanced Strength and Applied Stress Analysis*, 2nd ed., McGraw-Hill, New York, 1999. Copyright © 1999 The McGraw-Hill Companies.

Beam Cross-Sectional Shape	Factor C
Rectangular	1.2
Circular	1.11
Thin-walled tubular, round	2.00
Box sections[†]	1.00
Structural sections[†]	1.00

[†]Use area of web only.

EXAMPLE 4–6 Find the strain energy due to shear in a rectangular cross-section beam, simply supported, and having a uniformly distributed load.

Solution Using Appendix Table A–9–7, we find the shear force to be

$$V = \frac{wl}{2} - wx$$

Substituting into Eq. (4–19), with $C = 1.2$, gives

Answer

$$U = \frac{1.2}{2AG}\int_0^l \left(\frac{wl}{2} - wx\right)^2 dx = \frac{w^2 l^3}{20AG}$$

4-8 Example of Castigliano's Theorem

EXAMPLE 4–10 [illegible]