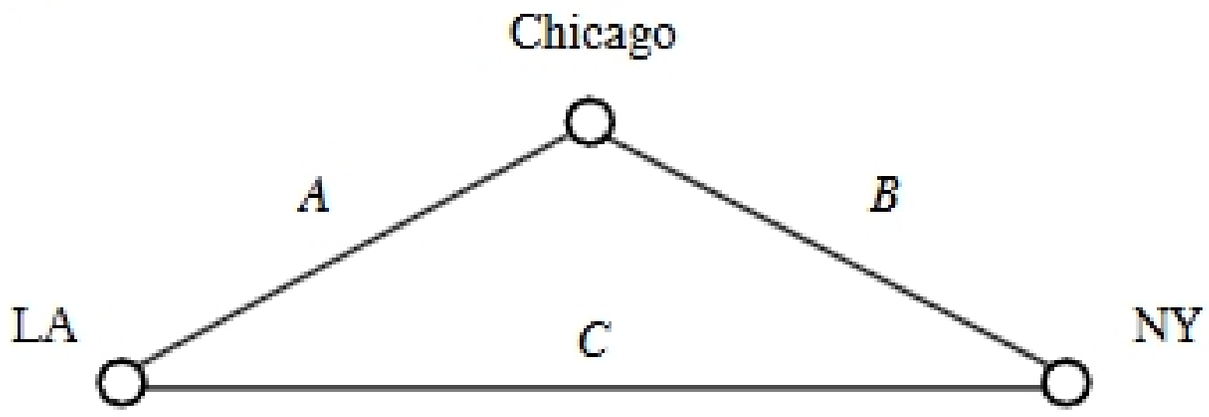
Probabilities in Transmission Links

Recall this network



Assume links independent, with

$$P(A) = a \quad P(B) = b \quad P(C) = c$$

Example 1

$$T_{LC} = A + BC = A + A'BC$$

$$P(T_{LC}) = P(A + BC) = P(A) + P(BC) - P(ABC) = a + bc - abc$$

or $$P(T_{LC}) = P(A + A'BC) = P(A) + P(A'BC) = a + bc - abc.$$

If $$\text{if} \quad a = b = c = \tfrac{1}{2} \qquad P(T_{LC}) = \tfrac{3}{8}.$$

Example 2

$$P(T_{LC} \mid T_{CN}) = \frac{P(T_{LC}T_{CN})}{P(T_{CN})}$$

$$P(T_{CN}) = P(B + AC) = b + ac - abc$$

$$T_{LC}T_{CN} = (A + BC)(B + AC) = AB + AC + BC$$

$$P(T_{LC}T_{CN}) = P(AB) + P(AC) + P(BC) - 2P(ABC)$$
$$= ab + ac + bc - 2abc$$

$$P(T_{LC} \mid T_{CN}) = \frac{ab + ac + bc - 2abc}{b + ac - abc}$$

$$\text{if} \quad a = b = c = \tfrac{1}{2} \qquad \text{then} \quad P(T_{LC} \mid T_{CN}) = \tfrac{4}{5}$$

$$\text{whereas} \quad P(T_{LC}) = \tfrac{3}{8}.$$

Example 3 (Bayes)

$$\text{Find} \quad P(A \mid T_{LC}) = \frac{P(AT_{LC})}{P(T_{LC})} = \frac{P(A)P(T_{LC} \mid A)}{P(A)P(T_{LC} \mid A) + P(A')P(T_{LC} \mid A')}$$

$$P(T_{LC} \mid A) = 1 \qquad P(T_{LC} \mid A') = bc$$

$$P(A \mid T_{LC}) = \frac{a}{a + (1-a)bc} = \frac{a}{a + bc - abc}$$

$$\text{if} \quad a = b = c = \tfrac{1}{2} \qquad \text{then} \quad P(A \mid T_{LC}) = \tfrac{4}{5}$$

$$\text{whereas} \quad P(A) = a = \tfrac{1}{2}.$$