

Chapter 2 Probability

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2.1 – 2.3

Classical Probability Concept

Assume all possible outcomes are equally likely, and if there are n equally likely ways for an event to occur out of totally N equally likely possible outcomes (Sample Space) then the probability of the event to occur is

$$\frac{n}{N}$$

Therefore counting is important.

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Sample Space and Probability

- ◆ **Random Experiment:** (Probability Experiment) an experiment whose outcomes depend on chance.
- ◆ **Sample Space (S):** collection of all possible outcomes in random experiment.
- ◆ **Event (E):** a collection of outcomes

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Sample Space and Event

- ◆ **Sample Space:** *Finite, Countable, Discrete*
 $S = \{\text{Head, Tail}\}$
 $S = \{\text{Life span of a human}\} = \{x \mid x \geq 0, x \in \mathbb{R}\}$
- ◆ **Event:** *Continuous*
 $E = \{\text{Head}\}$
 $E = \{\text{Life span of a human is less than 3 years}\}$

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Sample Space (Two Dice)

Sample Space:

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Sample point

$$S = \{(x, y) \mid x = 1, 2, 3, 4, 5, 6; y = 1, 2, 3, 4, 5, 6\}$$

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Notations

- $A = \emptyset \Rightarrow A$ is an empty set
- $A \subset B \Rightarrow A$ is a subset of B
- $A \supset B \Rightarrow B$ is a subset of A
- $A \cup B \Rightarrow$ Union of A & B
- $A \cap B \Rightarrow$ Intersection of A & B
- $A' \Rightarrow$ Complement of A

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Probability

$S = \{1, 2, 3, 4, 5, 6\}$
 $A = \{1, 2, 3\}$
 $B = \{3, 6\}$



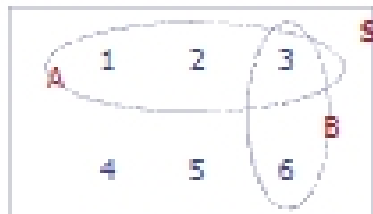
Venn Diagram
(with elements listed)

Intersection of events:
 $A \cap B \iff A \text{ and } B$
 Example: $A \cap B = \{3\}$

Union of events:
 $A \cup B \iff A \text{ or } B$
 Example: $A \cup B = \{1, 2, 3, 6\}$

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$S = \{1, 2, 3, 4, 5, 6\}$
 $A = \{1, 2, 3\}$
 $B = \{3, 6\}$



Venn Diagram
(with elements listed)


Complement of A: A'
 Example: $A' = \{4, 5, 6\}$

Subset: if all the elements in C are in A then $C \subset A$.
 Example: $C = \{1, 2\} \subset A$

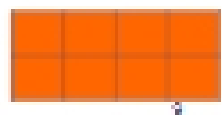
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Mutually Exclusive and Exhaustive

If A_1, A_2, \dots, A_k are events and $A_i \cap A_j = \emptyset$ for all $i \neq j$ then A_1, A_2, \dots, A_k are said to be **mutually exclusive** events.



If A_1, A_2, \dots, A_k are events in the sample space S , then A_1, A_2, \dots, A_k **exhaustive events** if $A_1 \cup A_2 \cup \dots \cup A_k = S$.



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Some basic laws among sets

- ◆ Commutative laws:
 $E \cup F = F \cup E; \quad E \cap F = F \cap E$
- ◆ Associative laws:
 $(E \cup F) \cup G = E \cup (F \cup G); \quad (E \cap F) \cap G = E \cap (F \cap G)$
- ◆ Distributive laws:
 $(E \cup F) \cap G = (E \cap G) \cup (F \cap G);$
 $(E \cap F) \cup G = (E \cup G) \cap (F \cup G)$

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DeMorgan's Laws

i) $\left(\bigcup_{i=1}^n E_i \right)^c = \bigcap_{i=1}^n E_i^c$

ii) $\left(\bigcap_{i=1}^n E_i \right)^c = \bigcup_{i=1}^n E_i^c$

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2.4 Probability of An Event

Probability is a real-valued set function P , also called probability measure, that assigns to each event A in the sample space S a number $P(A)$, called the probability of event A , such that the following properties are satisfied:

- **Postulate 1:** $P(A) \geq 0$, probability is always a value between 0 and 1.
- **Postulate 2:** $P(S) = 1$, total probability (all outcomes together) equals 1.
- **Postulate 3:** If A_1, A_2, \dots, A_k are events and $A_i \cap A_j = \emptyset$ then $P(A_1 \cup A_2 \cup \dots \cup A_k) = P(A_1) + P(A_2) + \dots + P(A_k)$

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Probability

Probability in Discrete Sample Space

Theorem: If A is an event in a discrete sample space S, then P(A) equals the sum of the probabilities of the individual outcomes comprising A.

Example: When casting a balanced die, the probability of getting 3 or less is

$$P(1 \cup 2 \cup 3) = P(1) + P(2) + P(3) \\ = 1/6 + 1/6 + 1/6 = 1/2$$

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Example: If, for an experiment, O_1, O_2, O_3, \dots , is an infinite sequence of outcomes, is

$$P(O_i) = \left(\frac{1}{2}\right)^i, \text{ for } i = 1, 2, 3, \dots$$

a probability measure?

$$P(S) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \frac{1}{1 - \frac{1}{2}} = 1 \quad \text{Yes!}$$

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2.5 Basic Rules of Probability

- $P(A^c) = 1 - P(A)$
- $P(\emptyset) = 0$
- If $A \subset B$ then $P(A) \leq P(B)$
- For any event A, $0 \leq P(A) \leq 1$
- For any two events A and B in a sample space S,
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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Complementation Rule

If the chance of a randomly selected individual living in community A to have disease H is .001, what is the probability that this person does not have disease H?

$$P(\text{having disease H}) = .001$$

$$P(\text{not having disease H})$$

$$= 1 - P(\text{having disease H})$$

$$= 1 - 0.001$$

$$= \mathbf{0.999}$$

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Birthday Problem

In a group of randomly select 23 people, what is the probability that at least two people have the same birth date? (Assume there are 365 days in a year.)

P(at least two people have the same birth date)

Too hard !!!

$$= 1 - P(\text{everybody has different birth date})$$

$$= 1 - [365 \times 364 \times \dots \times (365 - 23 + 1)] / 365^{23}$$

$$\approx .5$$

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Addition Rules for Probability from Postulate 3

A and B are mutually exclusive (disjointed) events



Additive Rule: $P(A \cup B) = P(A) + P(B)$

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