CS 2710 Foundations of AI
Lecture 25

Learning probability distributions

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Density estimation

Data: $D = \{D_1, D_2, .., D_n\}$
$D_i = \mathbf{x}_i$ a vector of attribute values

Attributes:

- modeled by random variables $\mathbf{X} = \{X_1, X_2, \ldots, X_d\}$ with:
  - **Continuous values**
  - **Discrete values**

E.g. ***blood pressure*** with numerical values
or ***chest pain*** with discrete values
[no-pain, mild, moderate, strong]

Underlying true probability distribution:

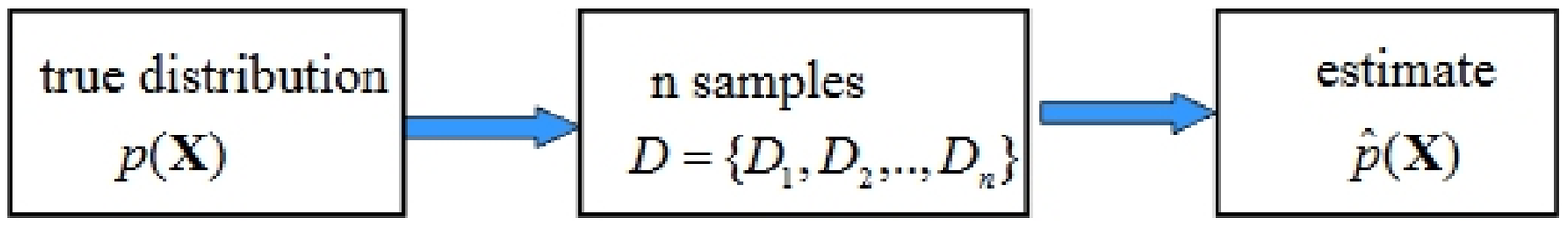
$p(\mathbf{X})$

Density estimation

Data: $D = \{D_1, D_2, .., D_n\}$

$D_i = \mathbf{x}_i$ a vector of attribute values

Objective: try to estimate the underlying true probability distribution over variables $\mathbf{X}$, $p(\mathbf{X})$, using examples in D



Standard (iid) assumptions: Samples

- **are independent of each other**
- **come from the same (identical) distribution (fixed $p(\mathbf{X})$)**

Learning via parameter estimation

In this lecture we consider **parametric density estimation**

Basic settings:

- A set of random variables $\mathbf{X} = \{X_1, X_2, \ldots, X_d\}$
- **A model of the distribution** over variables in X with parameters Θ
- **Data** $D = \{D_1, D_2, .., D_n\}$

Objective: find parameters $\hat{\Theta}$ that fit the data the best, or in other words reduce the misfit between the data and the model

- What is the best set of parameters?
  - There are various criteria one can apply here.

Parameter estimation. Basic criteria.

- **Maximum likelihood (ML) criterion**

$$\arg\max_{\Theta} p(D \mid \Theta, \xi)$$ ← Likelihood of data

ξ - represents prior (background) knowledge

- **Maximum a posteriori probability (MAP) criterion**

$$\arg\max_{\Theta} p(\Theta \mid D, \xi)$$ — Posterior probability

MAP selects the mode of the posterior

$$p(\Theta \mid D, \xi) = \frac{p(D \mid \Theta, \xi)\, p(\Theta \mid \xi)}{p(D \mid \xi)}$$

Parameter estimation. Coin example.

Coin example: we have a coin that can be biased

Outcomes: two possible values -- head or tail

Data: D a sequence of outcomes x_i such that

- **head** $x_i = 1$
- **tail** $x_i = 0$

Model: probability of a head θ

probability of a tail $(1-\theta)$

Objective:

We would like to estimate the probability of a **head** $\hat{\theta}$ from data