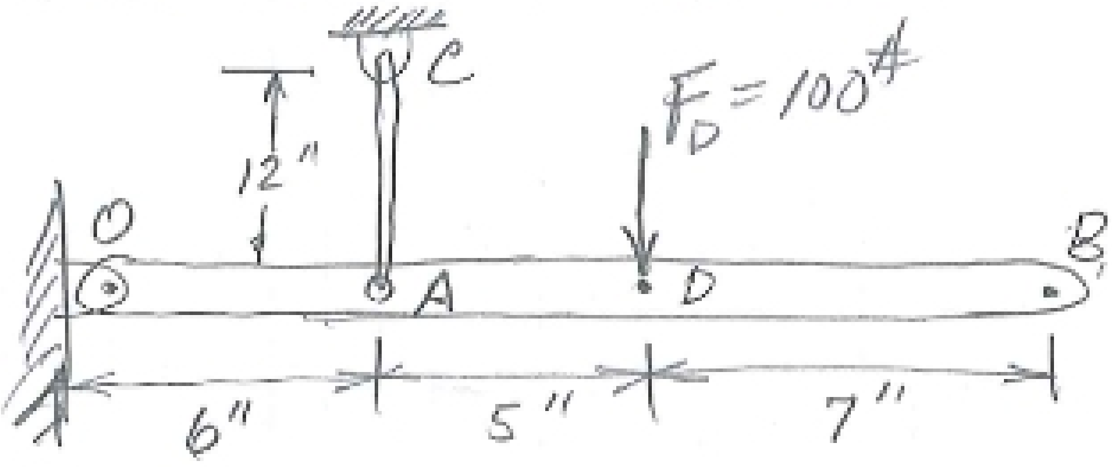
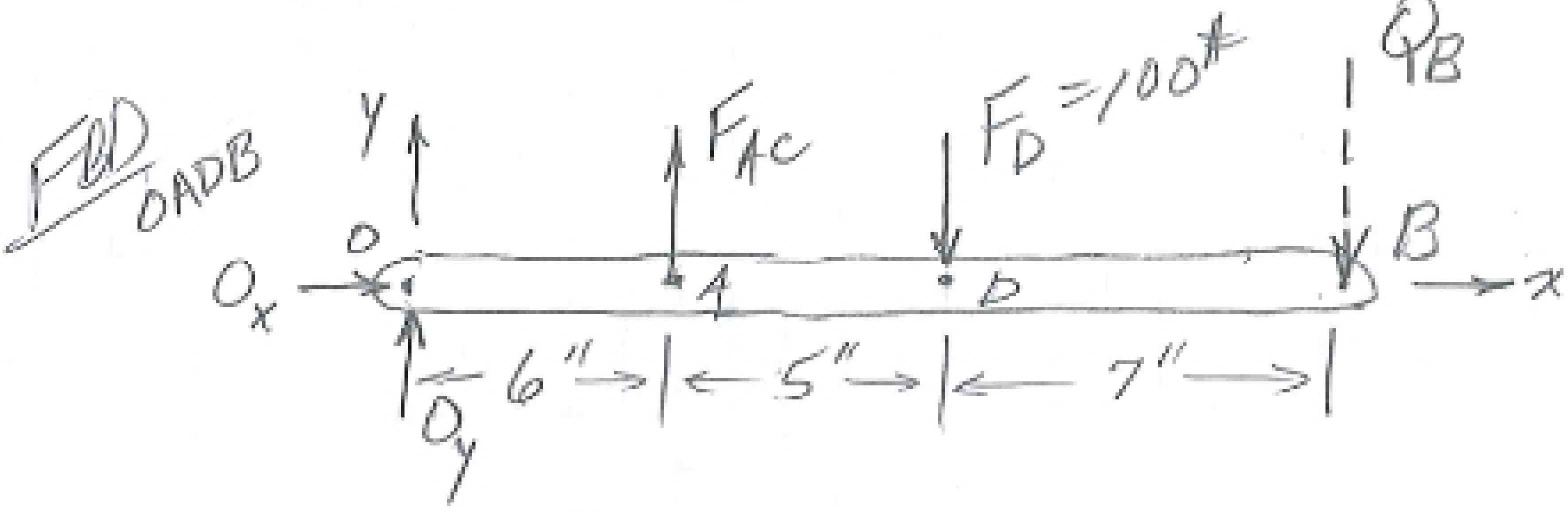
Prob 4-51*

Given: The structure loaded as shown. Member OB is $\frac{1}{4}$" thick aluminum plate with a rectangular cross-section (2" wide). Rod AC is steel, $\frac{1}{2}$" diameter, and has hooks formed at the ends.



Req'd: Find the vertical deflection at point B using Castigliano's Theorem. You may neglect shear.

Sol'n: need to add a "dummy load" vertical force at B



$$\Sigma F_x = 0 = \underline{O_x = 0}$$

$$\Sigma M_{O,z} = 0 = +6F_{AC} - 11F_D - 18Q_B$$

$$\therefore \underline{F_{AC}} = \frac{11}{6}(100\,lb) + 3Q_B = \underline{183.33 + 3Q_B}$$

$$\Sigma F_y = 0 = O_y + F_{AC} - F_D - Q_B$$

Prob 4-51* (cont'd)

$$O_y = F_D - F_{AC} + Q_B = 100 - (183.33 + 3Q_B) + Q_B$$

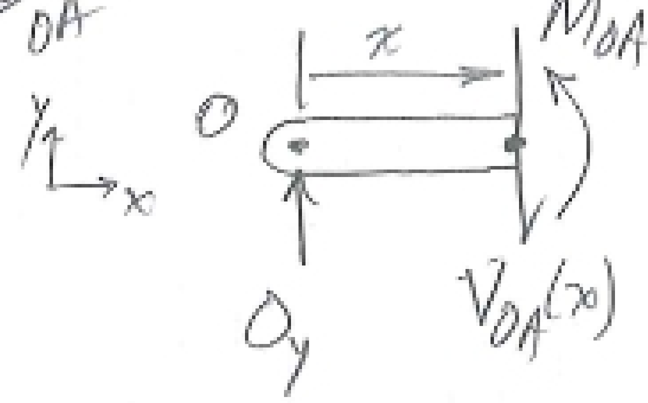
$$\therefore\ O_y = -2Q_B - 83.33$$

Castigliano's Theorem: $y_B = \frac{\partial U}{\partial Q_B} = \left(\frac{\partial U}{\partial Q_B}\right)_{\substack{beam\\OA\text{-}DB}} + \left(\frac{\partial U}{\partial Q_B}\right)_{\substack{bar\\AC}}$

$$y_B = \left(\frac{\partial U}{\partial Q_B}\right)_{OA} + \left(\frac{\partial U}{\partial Q_B}\right)_{AD} + \left(\frac{\partial U}{\partial Q_B}\right)_{DB} + \left(\frac{\partial U}{\partial Q_B}\right)_{AC}$$

(OA: bend, shear; AD: bend, shear; DB: bend, shear; AC: axial)

FBD_{OA}



$$\Sigma F_y = 0 = O_y - V_{OA}(x)$$

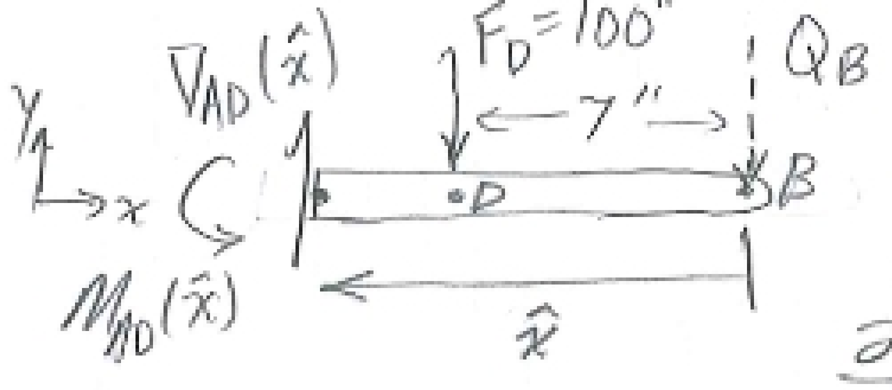
$$V_{OA}(x) = O_y = -2Q_B - 83.33$$

$$\frac{\partial V_{OA}}{\partial Q_B} = -2$$

$$\Sigma M_{z,cut} = 0 = +M_{OA}(x) - O_y x$$

$$M_{OA}(x) = O_y x = -2Q_B x - 83.33x;\quad \frac{\partial M_{OA}}{\partial Q_B} = -2x$$

FBD_{AD}



$$\Sigma F_y = 0 = V_{AD}(\hat{x}) - F_D - Q_B$$

$$V_{AD}(\hat{x}) = F_D + Q_B = 100 + Q_B$$

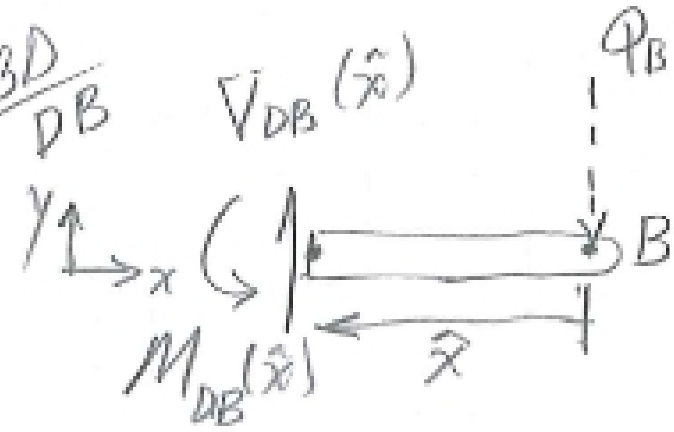
$$\frac{\partial V_{AD}}{\partial Q_B} = 1$$

$$\Sigma M_{z,cut} = 0 = M_{AD}(\hat{x}) - F_D(\hat{x} - 7) - Q_B \hat{x}$$

$$\therefore M_{AD}(\hat{x}) = 100\hat{x} - 700 + Q_B \hat{x};\quad \frac{\partial M_{AD}}{\partial Q_B} = \hat{x}$$

Prob 4-51* (cont'd)

FBD/DB



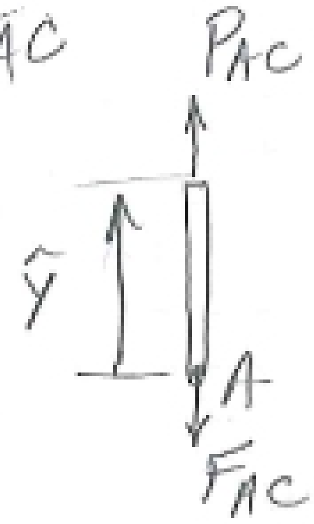
$$\Sigma F_y = 0 = V_{DB}(\hat{x}) - Q_B$$

$$V_{DB}(\hat{x}) = Q_B \; ; \; \frac{\partial V_{DB}}{\partial Q_B} = 1$$

$$\overset{\curvearrowleft +}{\Sigma} M_{z,cut} = 0 = M_{DB}(\hat{x}) - Q_B \hat{x}$$

$$\therefore M_{DB}(\hat{x}) = Q_B \hat{x} \; ; \; \frac{\partial M_{DB}}{\partial Q_B} = \hat{x}$$

FBD/AC



$$\Sigma F_y = 0 = P_{AC} - F_{AC}$$

$$\therefore P_{AC} = F_{AC} = 183.33 + 3 Q_B$$

$$\frac{\partial P_{AC}}{\partial Q_B} = 3$$

Now, neglecting shear:

$$y_B = \int_0^{l_{OA}} \frac{M_{OA} \frac{\partial M_{OA}}{\partial Q_B} dx}{(EI)_{OA}} + \int_{l_{DB}}^{l_{AB}} \frac{M_{AD} \frac{\partial M_{AD}}{\partial Q_B} d\hat{x}}{(EI)_{AD}}$$

$$+ \int_0^{l_{DB}} \frac{M_{DB} \frac{\partial M_{DB}}{\partial Q_B} d\hat{x}}{(EI)_{DB}} + \int_0^{l_{AC}} \frac{P_{AC} \frac{\partial P_{AC}}{\partial Q_B} d\hat{y}}{(EA)_{AC}}$$