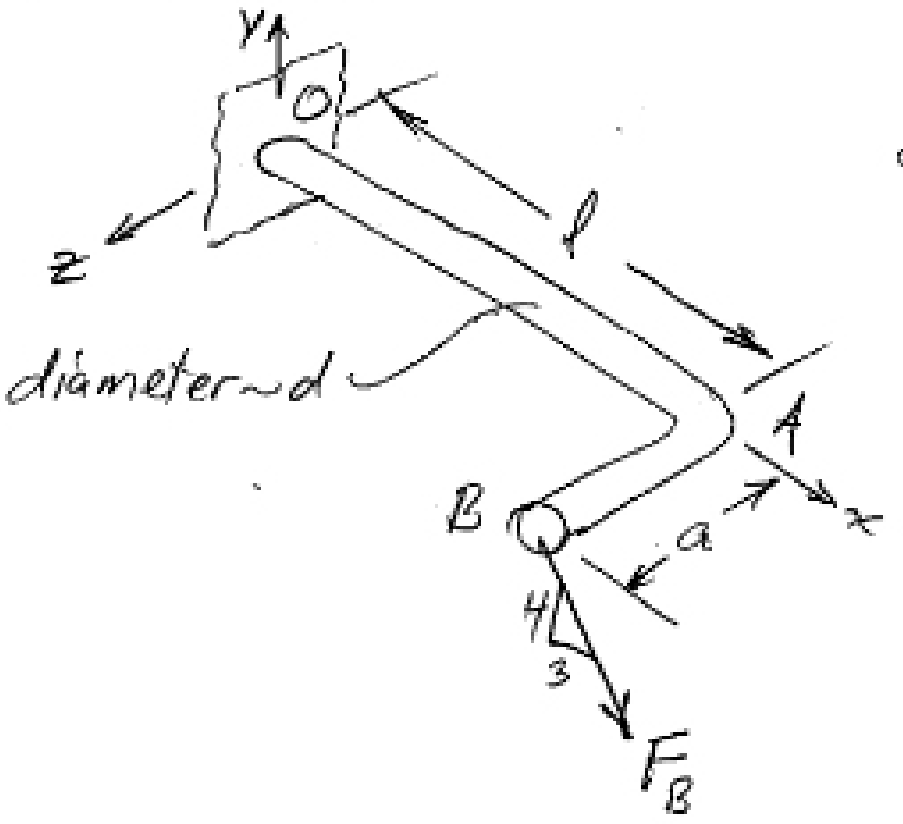
Prob 4-70



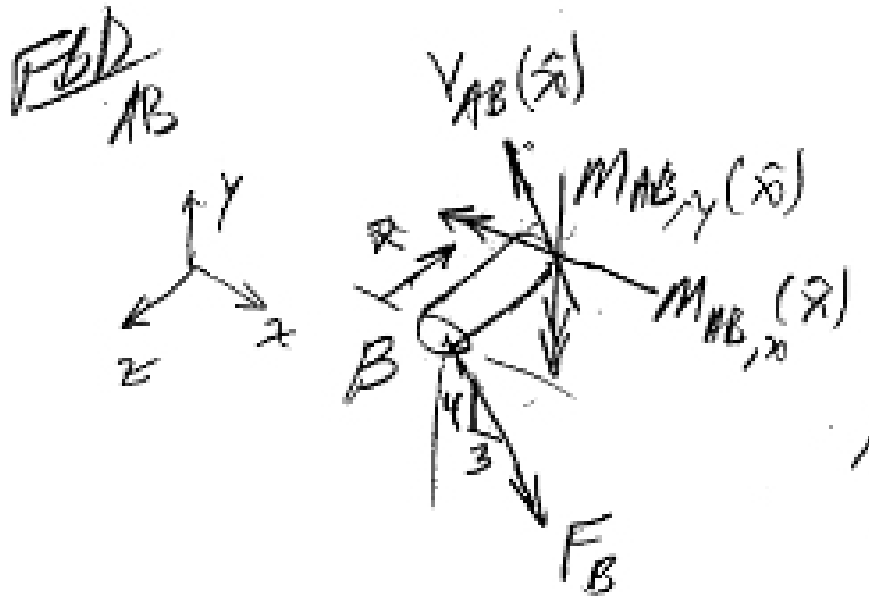
$$\delta_B = \frac{\partial U}{\partial F_B} = \left(\frac{\partial U}{\partial F_B}\right)_{AB} + \left(\frac{\partial U}{\partial F_B}\right)_{OA}$$

AB: bend + shear → neg.

OA: bend + torsion + axial → neg. ← shear

Both sections (OA & AB) appear to be "slender" enough to warrant neglecting shear.

FBD AB



$$V_{AB}(\hat{x}) = F_B, \quad \frac{\partial V_{AB}}{\partial F_B} = 1$$

$$\Sigma M_{x,cut} = 0 = -M_{AB,x}(\hat{x}) + \frac{4}{5} F_B \hat{x}$$

$$\underline{M_{AB,x}(\hat{x}) = \frac{4}{5} F_B \hat{x}}, \quad \underline{\frac{\partial M_{AB,x}}{\partial F_B} = \frac{4}{5}\hat{x}}$$

$$\Sigma M_{y,cut} = 0 = -M_{AB,y}(\hat{x}) + \frac{3}{5} F_B \hat{x} \Rightarrow \underline{M_{AB,y}(\hat{x}) = \frac{3}{5} F_B \hat{x}}$$

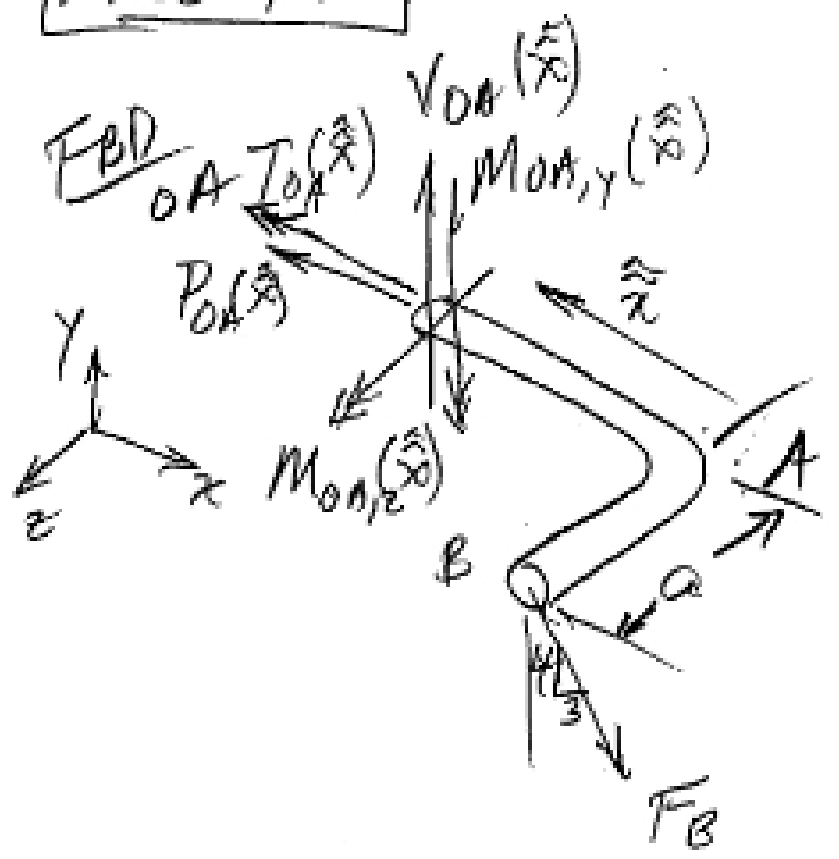
$$\underline{\frac{\partial M_{AB,y}}{\partial F_B} = \frac{3}{5}\hat{x}}$$

$$\underline{\left(\frac{\partial U}{\partial F_B}\right)_{AB}} = \int_0^a \frac{\left(\frac{4}{5} F_B \hat{x}\right)\left(\frac{4}{5}\hat{x}\right) d\hat{x}}{EI} + \int_0^a \frac{\left(\frac{3}{5} F_B \hat{x}\right)\left(\frac{3}{5}\hat{x}\right) d\hat{x}}{EI}$$

$$= \frac{16 F_B a^3}{75 EI} + \frac{9 F_B a^3}{75 EI} = \underline{\frac{F_B a^3}{3EI}}$$

(cont'd)

Prob 4-70



$$\Sigma F_y = 0 = V_{OA}(\hat{x}) - \tfrac{4}{5} F_B$$

$$\underline{V_{OA}(\hat{x}) = \tfrac{4}{5} F_B}\,; \quad \underline{\frac{\partial V_{OA}}{\partial F_B} = \frac{4}{5}}$$

$$\Sigma F_x = 0 = -P_{OA}(\hat{x}) + \tfrac{3}{5} F_B$$

$$\underline{P_{OA}(\hat{x}) = \tfrac{3}{5} F_B}\,; \quad \underline{\frac{\partial P_{OA}}{\partial F_B} = \frac{3}{5}}$$

$$\Sigma M_{x_{cut}} = 0 = -T_{OA}(\hat{x}) + \tfrac{4}{5} F_B a$$

$$\therefore \underline{T_{OA}(\hat{x}) = \tfrac{4}{5} F_B a}\,; \quad \underline{\frac{\partial T_{OA}}{\partial F_B} = \frac{4}{5} a}$$

$$\Sigma M_{y,cut} = 0 = -M_{OA,y}(\hat{x}) + \tfrac{3}{5} F_B a$$

$$\therefore \underline{M_{OA,y}(\hat{x}) = \tfrac{3}{5} F_B a}\,; \quad \underline{\frac{\partial M_{OA,y}}{\partial F_B} = \frac{3}{5} a}$$

$$\Sigma M_{z_{cut}} = 0 = +M_{OA,z}(\hat{x}) - \tfrac{4}{5} F_B \hat{x}$$

$$\therefore \underline{M_{OA,z}(\hat{x}) = \tfrac{4}{5} F_B \hat{x}}\,; \quad \underline{\frac{\partial M_{OA,z}}{\partial F_B} = \frac{4}{5} \hat{x}}$$

$$\left(\frac{\partial y}{\partial F_B}\right)_{OA} = \int_0^\ell \frac{\left(\frac{3}{5} F_B\right)\left(\frac{3}{5}\right) d\hat{x}}{EA} + \int_0^\ell \frac{\left(\frac{4}{5} F_B a\right)\left(\frac{4}{5} a\right) d\hat{x}}{GJ}$$

$$+ \int_0^\ell \frac{\left(\frac{3}{5} F_B a\right)\left(\frac{3}{5} a\right) d\hat{x}}{EI} + \int_0^\ell \frac{\left(\frac{4}{5} F_B \hat{x}\right)\left(\frac{4}{5} \hat{x}\right) d\hat{x}}{EI}$$

Prob 4-70 (cont'd)

$$\left(\frac{\partial U}{\partial F_B}\right)_{OA} = \frac{9F_B\ell}{25EA} + \frac{16F_Ba^2\ell}{256J} + \frac{9F_Ba^2\ell}{25EI} + \frac{16F_B\ell^3}{75EI}$$

Thus,

$$\delta_B = \frac{F_Ba^3}{3EI} + \frac{9F_B\ell}{25EA} + \frac{16F_Ba^2\ell}{256J} + \frac{9F_Ba^2\ell}{25EI} + \frac{16F_B\ell^3}{75EI}$$

But, $A = \frac{\pi}{4}d^2$, $I = \frac{\pi}{64}d^4$, $J = \frac{\pi}{32}d^4$

$$\therefore \boxed{\delta_B = \frac{64F_Ba^3}{3\pi Ed^4} + \frac{36F_B\ell}{25\pi Ed^2} + \frac{512F_Ba^2\ell}{25\pi Gd^4} + \frac{576F_Ba^2\ell}{25\pi Ed^4} + \frac{1024F_B\ell^3}{75\pi Ed^4}}$$

(1) (2) (3) (4) (5)

(b.) terms:

(1) c.f. bending in AB due directly to F_B

(2) axial deformation of OA due to hor. component of F_B

(3) torsion in OA due to vertical component of F_B

(4) bending in OA (about y-axis) due to hor. component of F_B

(5) bending in OA (about z-axis) due to vert. component of F_B

Numerical values: $F_B = 15$ lb, $\ell = 15$ in, $a = 7$ in

$d = 0.5$ in, $E = 30\times10^6$ lb/in^2, $G = 11.5\times10^6$ lb/in^2

Calculations (not shown) give:

$$\delta_B = \underbrace{0.01863}_{(1)} + \underbrace{1.375\times10^{-5}}_{(2)} + \underbrace{0.1000}_{(3)} + \underbrace{0.0431}_{(4)} + \underbrace{0.11734}_{(5)}$$

6.7% 0.005% 35.8% 15.4% 42.0%

$\therefore \boxed{\delta_B = 0.2791 \text{ in}}$ in the direction of F_B