Prob 4-79*

$d = 70\text{mm} \Rightarrow \underline{R = 35\text{mm}}$

$\therefore \underline{\frac{R}{h} = \frac{35}{4.5} = 7.78}$

(rule-of-thumb $\frac{R}{h} > 5\text{-}10 \Rightarrow$ thin)

→ use "thin" approach anyway (as instructed)

(a.)

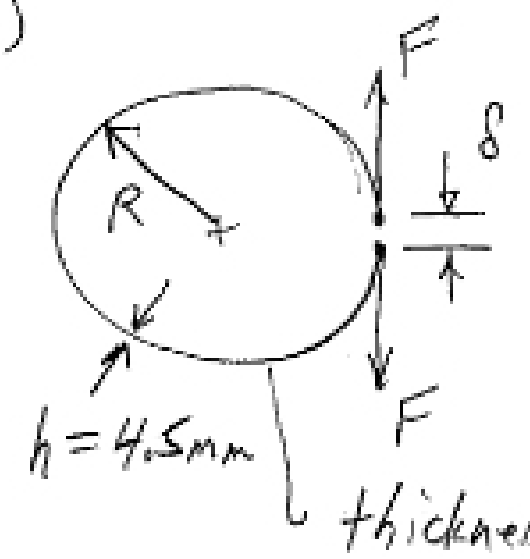
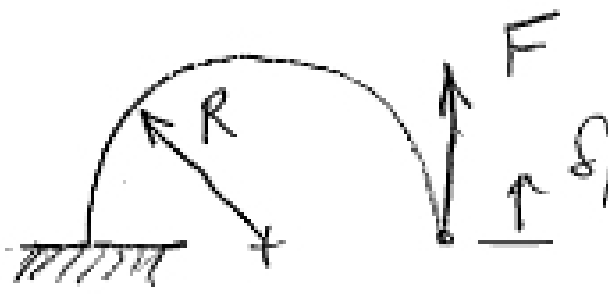


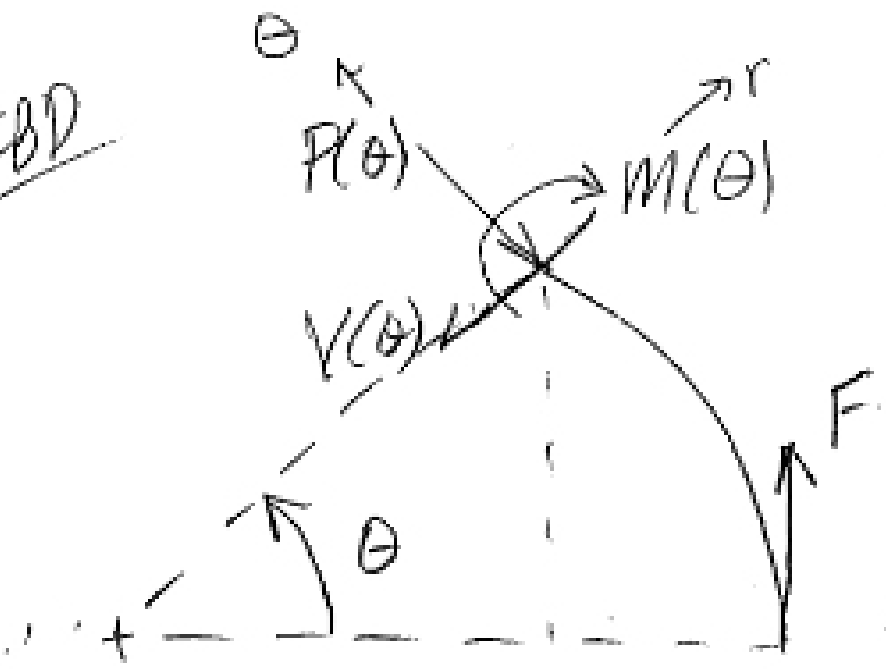
Table A-5, $E = 207\ \text{GPa}$, $G = 79.3\ \text{GPa}$

Use symmetry: $\delta = 2\delta_F$



$\delta_F = \frac{\partial U}{\partial F} = \left(\frac{\partial U}{\partial F}\right)_{\substack{\text{bend} \\ \text{axial} \\ \text{shear}}}$

FBD



$\sum F_r = 0 = -V(\theta) + F\sin\theta$

$\therefore \underline{V(\theta) = F\sin\theta}; \ \underline{\frac{\partial V}{\partial F} = \sin\theta}$

$\sum F_\theta = 0 = -P(\theta) + F\cos\theta$

$\therefore \underline{P(\theta) = F\cos\theta}; \ \underline{\frac{\partial P}{\partial F} = \cos\theta}$

$\sum M_{z,\text{cut}} = 0 = -M(\theta) + FR(1-\cos\theta)$

$\therefore \underline{M(\theta) = FR(1-\cos\theta)}; \ \underline{\frac{\partial M}{\partial F} = R(1-\cos\theta)}$

Prob 4-79 (cont'd)

$$\delta_I = \int_0^{\pi} \frac{[FR(1-\cos\theta)][R(1-\cos\theta)]\,Rd\theta}{EI} + \int_0^{\pi} \frac{[F\cos\theta][\cos\theta]\,Rd\theta}{EA}$$

$$+ C\int_0^{\pi} \frac{[F\sin\theta][\sin\theta]\,Rd\theta}{GA}$$

$$\delta_F = \int_0^{\pi} \frac{FR^3(1-2\cos\theta+\cos^2\theta)\,d\theta}{EI} + \int_0^{\pi} \frac{FR\cos^2\theta\,d\theta}{EA}$$

$$+ C\int_0^{\pi} \frac{FR\sin^2\theta\,d\theta}{GA}$$

$$\delta_F = \left(\frac{FR^3}{EI}\right)(\theta)\Big|_0^{\pi} - \left(\frac{2FR^3}{EI}\right)(\sin\theta)\Big|_0^{\pi} + \left(\frac{FR^3}{EI}\right)\left(\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi}$$

$$+ \left(\frac{FR}{EA}\right)\left(\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi} + \left(\frac{GFR}{GA}\right)\left(\frac{\theta}{2}-\frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi}$$

$$\delta_F = \frac{\pi FR^3}{EI} - (0) + \frac{\pi FR^3}{2EI} + \frac{\pi FR}{2EA} + \frac{\pi GFR}{2GA}$$

$$\delta_F = \frac{3\pi FR^3}{2EI} + \frac{\pi FR}{2EA} + \frac{\pi GFR}{2GA}$$

$$\therefore \delta = 2\delta_F = \frac{3\pi FR^3}{EI} + \frac{\pi FR}{EA} + \frac{\pi GFR}{GA}$$

(bend) (axial) (shear)

Prob 4-79* (cont'd)

$$\underline{I} = \frac{bh^3}{12} = \frac{(3mm)(4.5mm)^3}{12} = 22.78\,mm^4, \quad \underline{C = 1.2}\ (ref.)$$

$$\underline{A} = bh = (3mm)(4.5mm) = \underline{13.5\,mm^2}$$

So,

$$\delta = \frac{(3\pi)(35mm)^3 F}{\left(207\times10^3 \frac{N}{mm^2}\right)(22.78mm^4)} + \frac{(\pi)(35mm) F}{\left(207\times10^3 \frac{N}{mm^2}\right)(13.5mm^2)}$$

$$+ \frac{(1.2\pi)(35mm) F}{\left(79.3\times10^3 \frac{N}{mm^2}\right)(13.5mm^2)}$$

$$\delta = (0.085694 + 3.9347\times10^{-5} + 1.2325\times10^{-4}) F$$

$$\boxed{\delta = 0.085857\,F}$$

(b.) $\underline{\frac{R}{h} = \frac{35}{4.5} = 7.78}$

$$\delta_{\%shear} = \left(\frac{1.2325\times10^{-4} F}{0.085857F}\right)\times100 = \boxed{0.144\% = \delta_{\%\,shear}}$$

$$\delta_{\%axial} = \left(\frac{3.9347\times10^{-5} F}{0.085857F}\right)\times100 = \boxed{0.046\% = \delta_{\%\,axial}}$$

Note: Using "thick" curved beam equations (see text pp. 170-171) gives a deflection value of 0.08583 F. Thus, we overestimated by 0.03%!