

Class 26: Halting Problem

It is plain at any rate that the real mathematics (apart from the elements) has no *direct* utility in war. No one has yet found any war-like purpose to be served by the theory of numbers or relativity or quantum mechanics, and it seems very unlikely that anybody will do so for many years.

G. H. Hardy (Turing's advisor),
Mathematician's Apology, 1940

By 1943, Turing had used number theory to break the Nazi Enigma code.



Are there any undecidable problems?

The Proof-Finding Problem:

- Input: an axiomatic system, a statement S
- Output: If S is true, output a valid proof. If S is not true, output **false**.

Undecidable Problems

- We can prove a problem is undecidable by showing it is at least as hard as the proof-finding problem
- Here's a famous one:
 - Halting Problem
 - Input: a procedure P (described by a Scheme program) and its input I
 - Output: true if executing P on I halts (finishes execution), false otherwise.