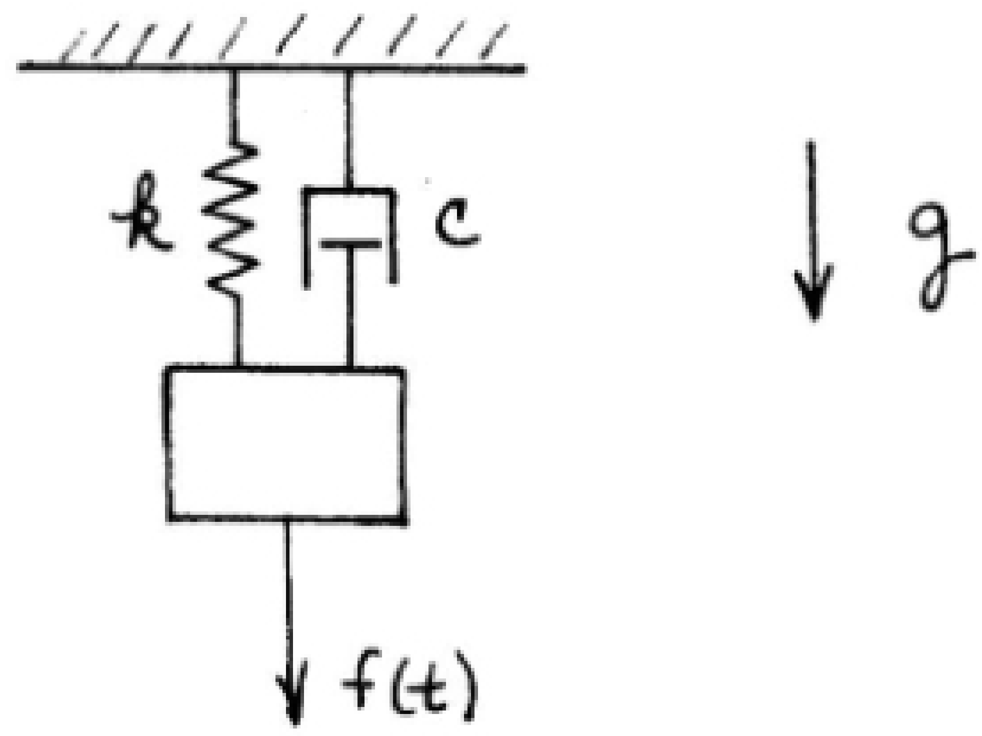
AAE 340 – Dynamics and Vibrations
Problem Set 4
Due: 9/18/13

Problem 1: In the system below, a particle of mass m can move in a vertical plane. An external force is continually applied to the system such that $f(t) = A_o\, m\cos(\Omega t)$. Define z as the position of the particle from its static equilibrium position. The system parameters are given as: $m = 2$ kg, $c = 0.5$ nt-s/met, $k = 2$ nt/met, $A_o = 10$ met/s^2, $\Omega = 0.14$ rad/s.



(a) Derive the system equation of motion in terms of the variable z.

$$\left\{ Ans: \quad \ddot{z} + \frac{c}{m}\dot{z} + \frac{k}{m}z = A_o \cos \Omega t \right\}$$

(b) Calculate the following quantities associated with the complimentary solution: undamped natural frequency, damped frequency, damping ratio, damped period (cycle time), characteristic roots, damping time constant.

(c) Determine the particular solution $z_p(t)$.

Determine the amplification factor. Consider that the damping ratio and the forcing frequency influence the amplitude of the steady-state response. Let $c = 0.5, 2, 5$ nt-s/met and determine the corresponding values of damping ratio.

The forcing frequency is given as 0.14 rad/s in this problem, but it could also vary. For each value of c, let the ratio $\dfrac{\Omega}{\omega_n}$ vary between 0 and 3. Plot the amplification factor as a function of the frequency ratio for the three values of damping. Put all 3 curves on the same figure. Mark the point that represents this problem.

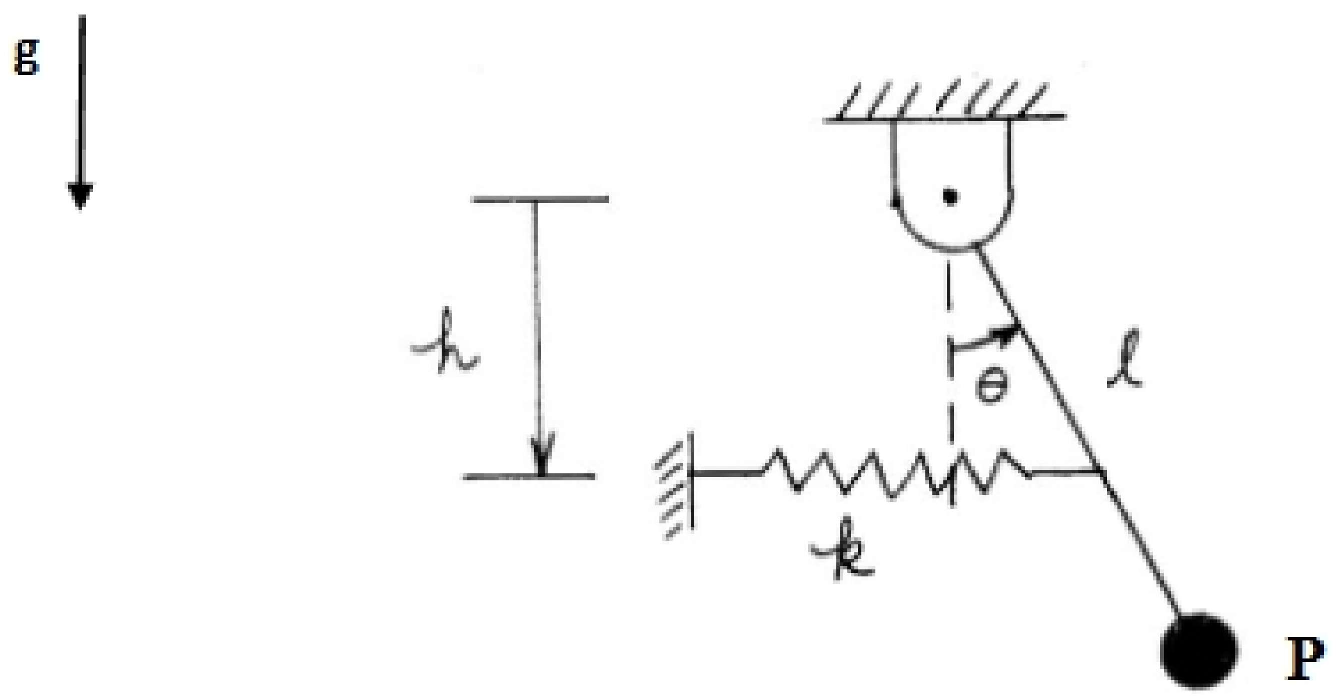
If the forcing frequency equals the natural frequency and the damping is very small, what happens to the amplification factor? What happens to the steady state amplitude?

(c) Determine the complete solution using the original values for c and Ω, i.e., $z(t)$. Let $z(0) = \dot{z}(0) = 0$.

(d) Plot the curves for the transient response, the steady-state response, and the total response for 75 sec; all the curves should appear on the same plot.

Mark the damping time constant on the transient and the total response curves. Is τ a good estimate for how long it takes for the forcing term to dominate the response? Why or why not?

Problem 2: A particle of mass m is attached to the end of a rigid rod of negligible mass. The rod is made to vibrate in the configuration indicated below. Assume that the spring is unstretched when $\theta = 0^{\circ}$.



(a) Derive the EOM that governs θ. (Assume small oscillations.)
$\left\{ Ans: m\ell^2\ddot{\theta} + (mg\ell + kh^2)\theta = 0 \right\}$

(b) Let $m = 1$ kg, $\ell = 1.5$ met, $k = 1$ nt/met, $h = 0.5$ met. Determine the characteristic roots, natural frequency, damped frequency, damping ratio, period.

(c) Let $\theta(0) = 13^{\circ}$, $\dot{\theta}(0) = 0^{\circ}$. Determine the total system response.

(d) If you wanted to increase the period of oscillation, would you shift the spring up or down? Why?
Shift the spring (in your selected direction) by .3 meters and compute the new period. Does this result make sense?