Last Name (Print): ____________________

First Name (Print): ____________________

ID number (Last 4 digits): __________

Section: ___

Submission deadlines:

- Turn in the written solutions by 4:00 pm on Tuesday September 16 in the homework slot outside 121 EE East.

Problem	Weight	Score
5	50	
6	50	
Total	100	

The solution submitted for grading represents my own analysis of the problem, and not that of another student.

Signature: ____________________

Neatly print the name(s) of the students you collaborated with on this assignment.

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Reading assignment:

- Franklin Chapters 1, 2 and Chapter 7, sections 7.1 though 7.3.
- Journal papers distributed in lecture

Problem 5: (50 points)

Figure 1 shows the armature circuit and free body diagram for the DC motor discussed in Lecture 2. Table 1 defines the variables and parameters, and provides the numeric values for the DC motor used in the Quanser modular servo system. Figure 2 shows the block diagram representation of the DC motor, where the transfer function of the

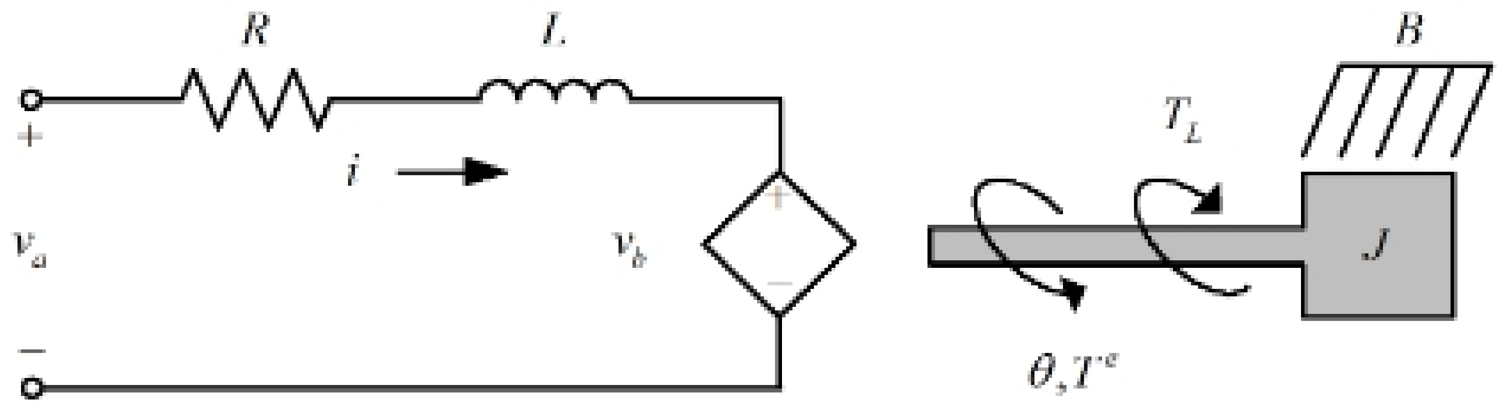

Figure 1: DC motor armature circuit and free body diagram.

Parameter	Symbol	Dimensions	Numerical Value
armature voltage	$v_a(t)$	V	
back EMF	$v_b(t)$	V	
armature current	i	A	
armature resistance	R	Ω	2.6
armature inductance	L	μH	180
torque constant	K_t	oz · in/A	1.088
back emf constant	K_e	mV/rpm	0.804
rotor angular velocity	$\omega(t)$	rad/s	
torque of electric origin	$T^e(t)$	oz · in	
load torque	$T_L(t)$	oz · in	
rotor moment of inertia	J	oz · in · s^2	5.523×10^{-5}
viscous frictional coefficient	B	oz · in/rad/s	3.249×10^{-3}

Table 1: DC motor parameters.

electrical subsystem, $G_E(s)$, and mechanical subsystem, $G_M(s)$ are

$$G_E(s) = \frac{K_t/R}{s(L/R)+1} = \frac{K_E}{s\tau_E+1} \quad \text{and} \quad G_M(s) = \frac{1/B}{s(J/B)+1} = \frac{K_M}{s\tau_M+1}.$$

Note that the DC gain K_E of the electrical subsystem transfer function is different from the back EMF constant K_e. The state space representation derived in lecture 2 is

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -R/L & -K_e/L \\ K_t/J & -B/J \end{pmatrix} x(t) + \begin{pmatrix} 1/L & 0 \\ 0 & -1/J \end{pmatrix} \begin{pmatrix} v_a \\ T_L \end{pmatrix} \\ \begin{pmatrix} \omega \\ i \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x(t). \end{aligned}$$

1. (10 points) When expressed in the same units, the torque conversion constant K_t must have the same numeric value as the back EMF constant K_e. This fact is proved using the principle of conservation of energy. The DC motor converts an instantaneous electrical power $p_e(t)$ into an instantaneous mechanical power $p_m(t)$. The instantaneous electrical power

$$p_e(t) = v_b(t)i(t)$$

is adsorbed by the dependent voltage source in Figure 1, and converted to an instantaneous mechanical power

$$p_m(t) = T^e(t)\omega(t)$$

that is delivered through the motor shaft. Conservation of energy demands that

$$p_e = p_m.$$

(a) (5 points) Using the fact that $p_e = p_m$, show that

$$K_e = K_t,$$

where K_e is the back EMF constant and K_t is the torque constant. Note that for this equality to hold, both K_e and K_t must be expressed in the same units.

(b) (5 points) Table 1 expresses the torque constant K_t in units of oz-in/A and the back EMF constant K_e in units of mV/rpm. Determine the numeric value of K_t in the SI units of N-m/A and K_e in the SI units of V/rad/s. Show that the SI units of N-m/A are identical to V/rad/s. Are the numeric values of K_t and K_e expressed in these units equal?

2. (15 points) In Lecture 2 we used the block diagram representation in Figure 2 to obtain the transfer functions

$$\begin{aligned}\frac{\Omega(s)}{V_a(s)} &= \frac{K_t/JL}{s^2 + s(B/J + R/L) + (RB + K_eK_t)/JL}\\ \frac{\Omega(s)}{T_L(s)} &= -\frac{S/J + R/JL}{s^2 + s(B/J + R/L) + (RB + K_eK_t)/JL}.\end{aligned}$$

Now you will determine these transfer functions from the state space representation. Using the input and output vectors

$$u(t) = \begin{pmatrix} v_a \\ T_L \end{pmatrix}, \quad \text{and} \quad y(t) = \begin{pmatrix} \omega \\ i \end{pmatrix},$$

determine the transfer function matrix

$$H(s) = C\,(sI - A)^{-1}\,B + D = \begin{pmatrix} \Omega/V_a & \Omega/T_L \\ I/V_a & I/T_L \end{pmatrix}$$

that relates $Y(s)$ and $U(s)$ as

$$Y(s) = H(s)U(s).$$

Do the transfer functions Ω/V_a and Ω/T_L obtained form the matrix $H(s)$ agree with those obtained in Lecture 2 from the block diagram representation?

3. (25 points) Using the fact that for a well designed DC motor the electrical time constant $\tau_E = L/R$ is significantly smaller than the mechanical time constant $\tau_M = J/B$, this problem shows how to approximate the second-order transfer

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_t/JL}{s^2 + s(B/J + R/L) + (RB + K_eK_t)/JL}$$

with the first-order transfer function

$$\frac{\Omega(s)}{V_a(s)} = \frac{K}{s\tau + 1}.$$

(a) (3 points) Determine the numeric value of τ_E, τ_M, and the ratio τ_M/τ_E using the numeric values provided in Table 1 for the Quanser DC motor.

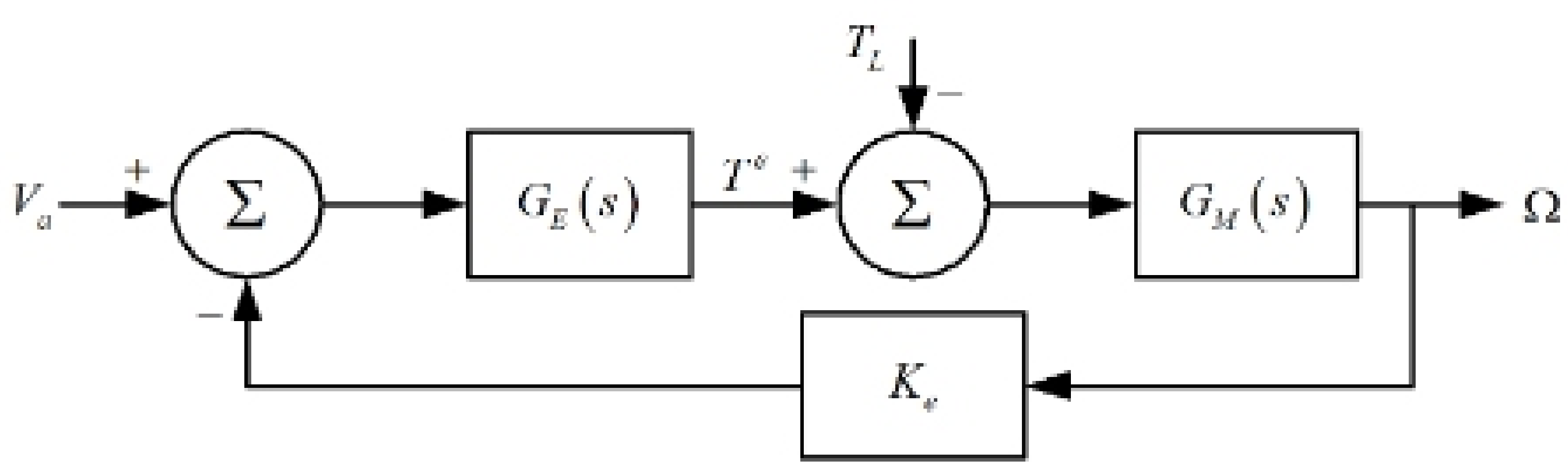

Figure 2: DC motor block diagram.