**EE 380** **Problem Set 3 Cover Sheet** **Fall 2014**

Last Name (Print): ____________________________

First Name (Print): ____________________________

ID number (Last 4 digits): ______________

Section: ___

Submission deadlines:

- Turn in the written solutions by 4:00 pm on Tuesday September 23 in the homework slot outside 121 EE East.

| Problem | Weight | Score |
|---|---|---|
| 7 | 25 | |
| 8 | 30 | |
| 9 | 30 | |
| 10 | 15 | |
| Total | 100 | |

The solution submitted for grading represents my own analysis of the problem, and not that of another student.

Signature: ____________________________

Neatly print the name(s) of the students you collaborated with on this assignment.

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**Problem 7** (25 points)

A dynamic system with input $u(t)$ and output $y(t)$ has the transfer function representation

$$G_p(s) = \frac{2s-1}{s^4+6s^3+4s^2+2s+1}.$$

1. (5 points) Determine the ODE representation of the system and express your answer in the standard form

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_o y = b_m \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt^{m-1}} + \cdots + b_o u.$$

2. (12 points) Obtain an all-integrator block diagram representation by expressing the transfer function as

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{P(s)}{U(s)}\frac{Y(s)}{P(s)} = \frac{1}{d(s)}\frac{n(s)}{1},$$

where $n(s)$ and $d(s)$ are polynomials in $s$.

3. (8 points) Find a state-space representation for the system using the all-integrator block diagram derived in part 2 along with the state variable assignments $x_1 = p, x_2 = \dot{x}_1(t), \ldots, x_n = \dot{x}_{n-1}(t)$. This assignment places the state space representation in *controllable canonical form*.

**Problem 8:** (30 points)

Consider a LTI SISO system with the ODE representation

$$\ddot{y} + 5\dot{y} + 6y = 4u \tag{1}$$

1. (8 points) Using Laplace transform methods, calculate the zero-input response $y_{zi(t)}$ and the zero-state response $y_{zs(t)}$ for the initial conditions $y(0) = 1, \dot{y}(0) = -1$ and the input $u(t) = e^{-t}u_o(t)$, where $u_o(t)$ is the unit-step function.
2. (5 points) Following the result in problem 7, find the controllable canonical state-space representation. For this case, where $m = 0$, let $P(s)/U(s) = n(s)/d(s)$, $P(s)/Y(s) = 1$, and use the state-variable assignment $x_1 = p = y$ and $x_2 = \dot{x}_1$.
3. (2 points) Determine the initial state vector $x(0)$.
4. (5 points) Compute the state transition matrix $\Phi(t)$. Check your answer by showing that $\Phi(0) = I$.
5. (10 points) Using the state-transition matrix, calculate the zero-input response $y_{zi(t)}$ and the zero-state response $y_{zs(t)}$ for the initial conditions and the input provided in part 1.

**Problem 9:** (30 points)

A single-input single-output (SISO) system with input $u(t)$ and output $y(t)$ is represented by the state space model

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) \\ y(t) &= \begin{pmatrix} 0 & -1 \end{pmatrix} x(t) + u(t). \end{aligned}$$

1. (5 points) Determine the eigenvalues of the system matrix $A$.
2. (5 points) Determine the state transition matrix $\phi(t)$ and check your answer by verifying that $\phi(0) = I$.
3. (10 points) Calculate the zero-state unit-step response of the system by applying the following result from lecture 5

$$y(t) = C\phi(t)x(0) + C\int_0^t \phi(t-\tau)Bu(\tau)d\tau + Du(t).$$

4. (6 points) Determine the transfer function representation of the system using the $A$, $B$, $C$ and $D$ matrices. If possible, cancel poles and zeros with the same location in the $s$-plane. Place your result in the standard form

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + \cdots + b_1 s + b_o}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$

5. (4 points) Using the transfer function obtained in part 4, determine the zero-state unit-step response and check your answer against that obtained in part 3.

**Problem 10:** (15 points)

1. (3 points) Suppose a single-input single-output (SISO) system is represented by

$$\begin{aligned} \dot{x} &= Fx + Gu \\ y &= Hx. \end{aligned}$$

We can find another state-space representation by introducing a new state vector $\bar{x}$ that is related to the original state vector $x$ by $x = P\bar{x}$, where $P$ is a constant, nonsingular matrix (a matrix is nonsingular if its inverse exists). In terms of the new state vector $\bar{x}$, the SISO system representation is

$$\begin{aligned} \dot{\bar{x}} &= \bar{F}\bar{x} + \bar{G}u \\ y &= \bar{H}\bar{x}. \end{aligned}$$

Express the matrices $\bar{F}$, $\bar{G}$ and $\bar{H}$ in terms of $F$, $G$ $H$, $P$ and $P^{-1}$. If $x$ is a $n \times 1$ column vector, what must be the dimensions of the matrix $P$?

2. (1 point) Given that the only restriction on the transformation matrix $P$ is that it is nonsingular, how many different state-space representation exist for a given system?

3. (5 points) Consider the state-space representation

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 6 \\ 0 & 2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x. \end{aligned}$$

Determine the eigenvalues of the system matrix and show that the corresponding eigenvectors are

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

4. (6 points) Apply the transformation in part 1 to the state-space system in part 3 using

$$P = \begin{pmatrix} v_1 & v_2 \end{pmatrix}.$$

Compute the matrices $\bar{F}$, $\bar{G}$ and $\bar{H}$ in the new state-space representation

$$\begin{aligned} \dot{\bar{x}} &= \bar{F}\bar{x} + \bar{G}u \\ y &= \bar{H}\bar{x}. \end{aligned}$$

What do the numbers on the main diagonal of $\bar{F}$ represent?