EE 380 **Problem Set 4 Cover Sheet** **Fall 2014**

Last Name (Print): ___________________________

First Name (Print): ___________________________

ID number (Last 4 digits): ______________

Section: ___

Submission deadlines:

- Turn in the written solutions by 4:00 pm on Tuesday September 30 in the homework slot outside 121 EE East.

Problem	Weight	Score
11	25	
12	25	
13	25	
14	25	
Total	100	

The solution submitted for grading represents my own analysis of the problem, and not that of another student.

Signature: ___________________________

Neatly print the name(s) of the students you collaborated with on this assignment.

Reading assignment: Chapters 3 and Chapter 7, section 7.4

Problem 11 (25 points)

1. (18 points) Consider a second-order system with initial state $x(0)$ and state space representation

$$\dot{x} = Fx.$$

Suppose that the 2×2 system matrix F has eigenvalues λ_1 and λ_2 with associated eigenvectors v_1 and v_2. Using the matrix

$$P = \begin{pmatrix} v_1 & v_2 \end{pmatrix},$$

in problem set 3 problem 10 you showed that

$$\bar{F} = P^{-1}FP = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}.$$

(a) (10 points) Using the definition

$$e^{Ft} = \sum_{k=0}^{\infty} \frac{t^k}{k!} F^k,$$

show that

$$e^{Ft} = Pe^{\bar{F}t}P^{-1} = P \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix} P^{-1}.$$

(b) (8 points) Using the result in part (1), show that the zero-input solution to the state space equation

$$\dot{x} = Fx(t)$$

with initial state

$$x(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix}$$

can be expressed as

$$x(t) = v_1 \, e^{\lambda_1 t} \, \bar{x}_1(0) + v_2 \, e^{\lambda_2 t} \, \bar{x}_2(0) \qquad (1)$$

for $t \geq 0$, where

$$\bar{x}(0) = \begin{pmatrix} \bar{x}_1(0) \\ \bar{x}_2(0) \end{pmatrix} = P^{-1}x(0).$$

Equation (1) shows that the solution $x(t)$ is a weighted sum of the eigenvectors of the system matrix and that the weighting coefficients are determined by the system modes $e^{\lambda_i t}$. For this reason, equation (1) is known as the modal decomposition of $x(t)$.

2. (7 points) Once again consider the system in Problem 10 with $u(t) = 0$ for all $t \geq 0$

$$\dot{x} = \begin{pmatrix} -1 & 6 \\ 0 & 2 \end{pmatrix} x = Fx,$$

and the initial state vector

$$x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Determine the modal decomposition of the solution $x(t)$ using equation (1).

Problem 12: (25 points)

A continuous-time SISO system is described by the state-space model

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} \alpha & 1 \end{pmatrix} x \end{aligned}$$

where α is a real-valued constant.

1. (5 points) Calculate the observability matrix Q.
2. (8 points) For what range of values α is the observability matrix Q nonsingular?
3. (5 points) Suppose that $\alpha = 1$, $u(t) = 0$ for $t \geq 0$, and you observe that $y(0) = 1$ and $\dot{y}(0) = -2$. Can you find a unique value for $x(0)$? If so, what is $x(0)$?
4. (7 points) In part 3 suppose that $\alpha = 0$. Specify all possible values of $x(0)$, if any, that satisfy

$$\begin{pmatrix} y(0) \\ \dot{y}(0) \end{pmatrix} = Qx(0).$$