

Last Name (Print): \_\_\_\_\_

First Name (Print): \_\_\_\_\_

ID number (Last 4 digits): \_\_\_\_\_

Section: \_\_\_\_\_

Submission deadlines:

- Turn in the written solutions by 4:00 pm on Friday November 14 in the homework slot outside 121 EE East.

Problem	Weight	Score
25	15	
26	25	
27	20	
28	20	
29	20	
Total	100	

The solution submitted for grading represents my own analysis of the problem, and not that of another student.

Signature: \_\_\_\_\_

Neatly print the name(s) of the students you collaborated with on this assignment.

_____	_____
_____	_____
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**Problem 25:** (15 points)

The root locus for the closed-loop system in Figure 1 is shown in Figure 2 as the proportional control gain  $G_c(s) = K_p$  is varied from zero towards infinity. The numerator and denominator of the plant transfer function  $G_p(s)$  are both monic polynomials. Answer the following questions using only the root locus provided in Figure 2. **Do not** use the Routh-Hurwitz criterion to answer questions regarding stability.

1. (3 points) Is the open-loop system BIBO stable?
2. (3 points) For what range of  $K_p \geq 0$ , if any, is the closed-loop system BIBO stable?
3. (3 points) Choose the largest value of  $K_p$  so that  $\zeta > 0.7071$  for the dominant complex conjugate poles.
4. (3 points) For the value of  $K_p$  chosen in part 3, what is the steady-state system error for a unit-step input?
5. (3 points) For the value of  $K_p$  chosen in part 3, estimate the peak-overshoot, rise-time, and settling time of the closed-loop response to a unit-step input. What factors, if any, may affect your estimated values?

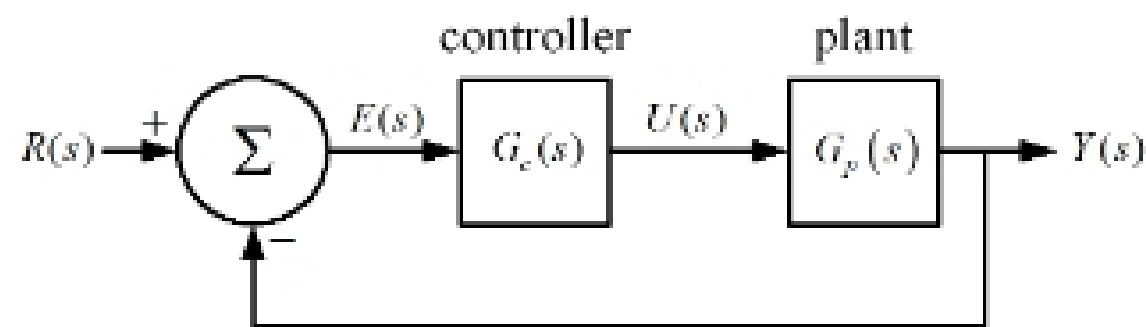


Figure 1: Closed-loop system with controller  $G_c(s)$ .

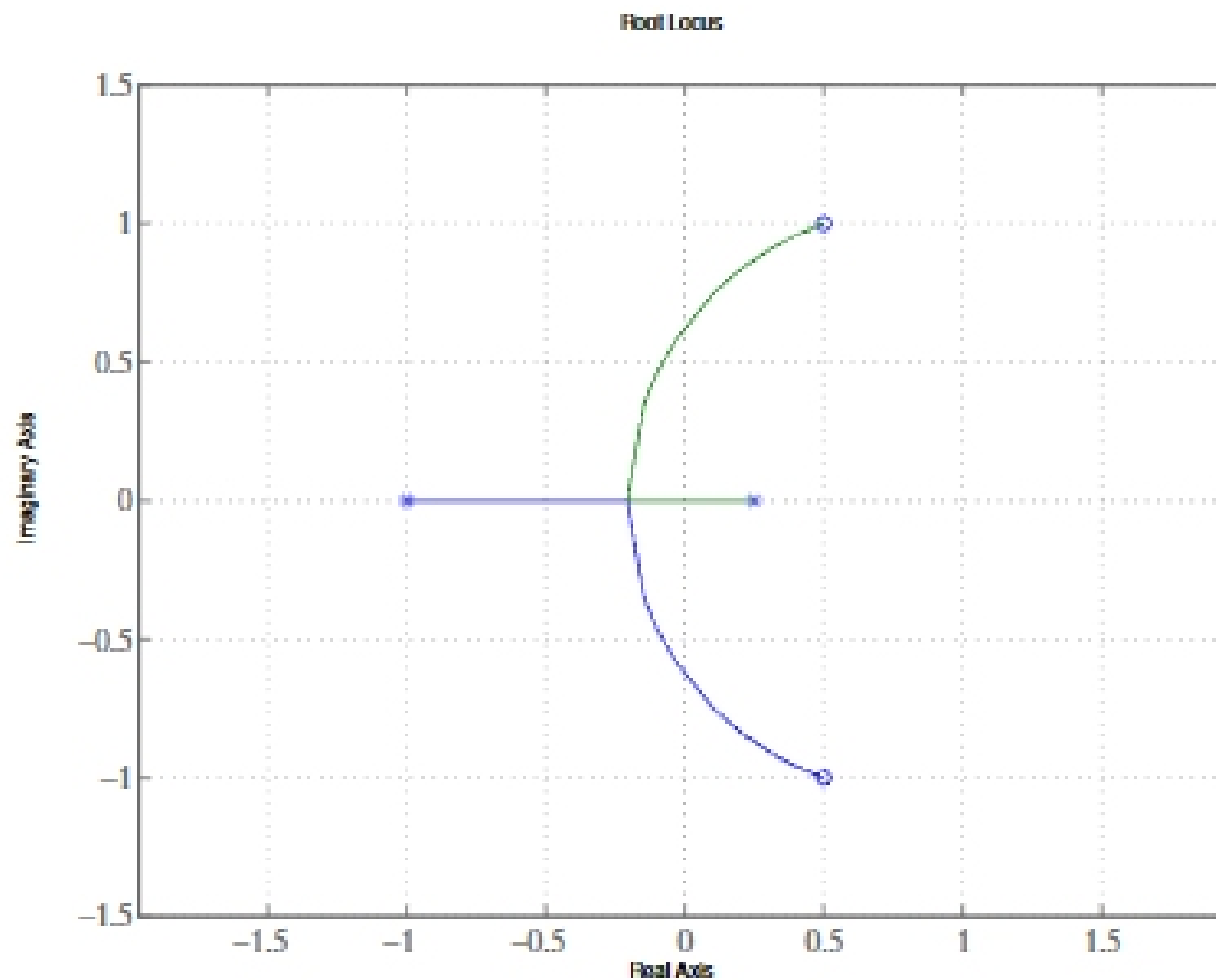


Figure 2: Root locus as the proportional gain  $K_p$  is varied from zero towards infinity.

**Problem 26:** (25 points)

Consider the closed-loop system in Figure 3.

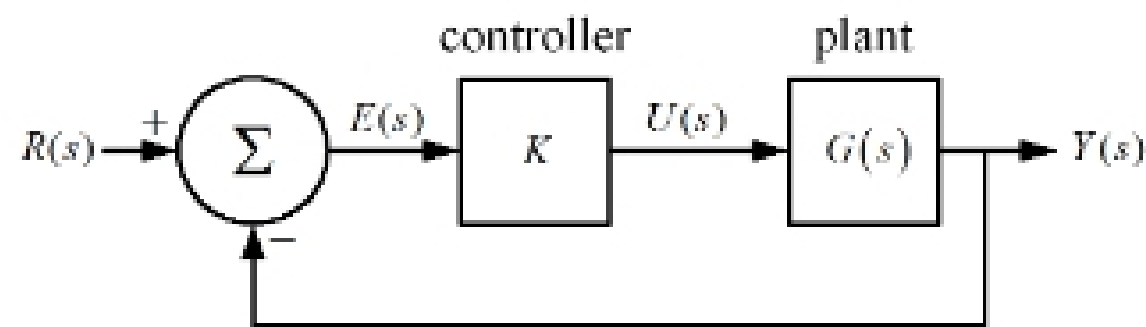


Figure 3: Feedback control system with proportional gain  $K$ .

For each system listed below:

- Neatly sketch *by hand* the root loci as the proportional gain is varied from zero towards infinity. Be sure to specify breakaway and breakin points, asymptotes, arrival and departure angles, and imaginary-axis crossings, if any. Check your answer using the MATLAB command `rlocus(num,den)`. Attach a copy of the MATLAB root locus plot along with your hand sketch.
- For  $K \geq 0$ , specify the range of  $K$ , if any, that the system is BIBO stable.

1. (5 points)  $G(s) = \frac{s^2 + 8s + 12}{s^2 + 8s + 25}$

2. (5 points)  $G(s) = \frac{s^2 + 2}{s^2 + 7s + 12}$

3. (5 points)  $G(s) = \frac{1}{(s+1)^3(s+4)}$

4. (5 points)  $G(s) = \frac{s^2 + 1}{s^3 + 4s}$

5. (5 points)  $G(s) = \frac{s+2}{s^4}$

**Problem 27:** (20 points)

Once again consider the closed-loop system in Figure 3.

1. (6 points) Suppose that

$$G(s) = \frac{(s + \alpha)}{s(s + 1)(s + 9)}$$

where  $\alpha$  is a design parameter. Set the controller gain  $K$  to unity and neatly sketch *by hand* the root loci of the closed-loop system as the parameter  $\alpha$  is varied from zero towards infinity. Specify the breakaway and breakin points, asymptotes, arrival and departure angles, and imaginary-axis crossings, if any.

2. (14 points) Suppose that

$$G(s) = \frac{s + 4}{(s + 1)(s + 2)(s + 3)}$$

- (a) (7 points) Neatly sketch *by hand* root loci for  $K > 0$ . Be sure to specify breakaway and breakin points, asymptotes, arrival and departure angles, and imaginary-axis crossings, if any. Check your answer using the MATLAB command `rlocus(num,den)`. Attach a copy of the MATLAB root locus plot along with your hand sketch.
- (b) (7 points) Neatly sketch *by hand* the complementary root assuming that  $K < 0$ . Be sure to specify breakaway and breakin points, asymptotes, arrival and departure angles, and imaginary-axis crossings, if any. Check your answer using the MATLAB command `rlocus(num,den)`. Attach a copy of the MATLAB root locus plot along with your hand sketch.