AAE 340 – Dynamics and Vibrations
Problem Set 1
Due: 8/28/13

On the 340 Blackboard Learn site, a review of dimensions is included in a document: *Supplementary Material → AAE 203 Review → Dim Analysis F13*. After reading the document, consider the following problems:

Problem 1: A student in a dynamics class derived the following equation to model the effects of air resistance on a vehicle:

$$y = \frac{bu + x}{\beta^3}\left(1 - Ce^{-u\tau}\right) - \frac{\tau x}{u} + e$$

Note that $\dim[e] = L$, $\dim[u] = 1/T$.

Determine the dimensions of each quantity for which the dimensions are not already defined.

Problem 2: The following differential equation governs the motion in a one-particle system,

$$2\frac{d^2 p}{dt^2} - \eta\frac{d^2 a}{dt^2}\cos(a) + \eta\left(\frac{da}{dt}\right)^2 \sin(a) - 2g\sin 30^\circ = -\mu\left[2g\cos 30^\circ - \eta\frac{d^2 a}{dt^2}\sin(a) + \eta\left(\frac{da}{dt}\right)^2\cos(a)\right]$$

Assume that g is the gravity constant. Determine the dimensions of $p,\ \eta,\ a,\ \mu$.
Is the complete equation dimensionally consistent? How do you know?

Problem 3: Consider the following differential equation,

$$\frac{d^2 x}{dy^2} - 2\Gamma r\frac{d\theta}{dy} - \frac{\Gamma^2\ Z\sin\theta}{m} = -\frac{\Gamma^2 Z^3}{mr^3}\left\{\frac{Gh}{m} + \frac{3m(\Gamma^2\cos\theta)k}{Z^2}\right\}$$

and $\Gamma = \int_0^y \left\{\left(\frac{\beta}{r}\right)\cos(\theta)\right\}dy$, $Z = \sum_{j=1}^{3} c_j\left(\frac{r\beta}{x}\right)^j$, $\int_0^x\int_0^u \left(x^2 + u^2\right)dx\,du = \frac{Gk}{m}$

Assume that the following quantities are defined with the following dimensions.

$\dim[y] = T$ $\qquad\qquad$ $\dim[r] = L$

$\dim[\Gamma] = \frac{1}{T}$ $\qquad\qquad$ $\dim[m] = M$

Determine the dimensions of $x,\ Z,\ u,\ \beta,\ c_1,\ c_2,\ c_3,\ k$.

Is the equation dimensionally correct? How do you know?

Problem 4: Two identical cubes, B and C, as well as a half-cube A (wedge) are oriented with respect to each other as indicated below. The boxes can rotate relative to each other. Fixed in each is a set of unit vectors defined parallel to the edges. The wedge also includes a right-handed set of unit vectors $\hat{s}_1, \hat{s}_2, \hat{s}_3$ such that $\hat{s}_2$ is parallel to the inclined surface. Each cube is square with edge length L.

(a) Each set of unit vectors is right-handed. For each of the 4 sets of unit vectors, add the appropriate third unit vector to the figure.

(b) Define unit vectors $\hat{b}_1, \hat{b}_2, \hat{b}_3$ in terms of the unit vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$.
Define $\hat{c}_1, \hat{c}_2, \hat{c}_3$ in terms of $\hat{a}_1, \hat{a}_2, \hat{a}_3$.
Define $\hat{a}_1, \hat{a}_2, \hat{a}_3$ in terms of $\hat{s}_1, \hat{s}_2, \hat{s}_3$.

(c) Write the position vector $\bar{r}^{QR}$ and express it in terms of $\hat{b}_1, \hat{b}_2, \hat{b}_3$.
Is ${}^b\bar{v}^{QR}$ generic? Why or why not?
Is ${}^b\bar{v}^{QR} = {}^b\bar{v}^R$? Justify your answer.

(d) Assuming that you are interested in ${}^s\bar{v}^R$, select an appropriate base point for the position vector.
Derive an expression for ${}^s\bar{v}^R$ in terms of L, ϕ, γ and the rates of change of the angles, but express it in terms of $\hat{b}_1, \hat{b}_2, \hat{b}_3$.