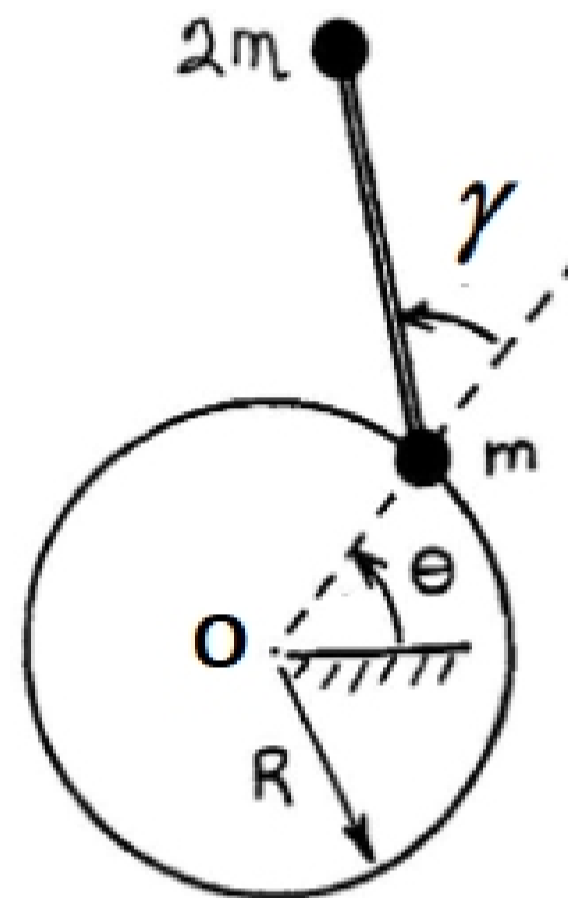


**AAE 340 – Dynamics and Vibrations**

**Problem Set 7**

**Due: 10/16/13**

**Problem 1:** Two particles (mass  $m$  and  $2m$ ) are connected to a massless rod of length  $L$ . The particles move on a smooth, *horizontal* surface. The motion of one particle is constrained to a fixed, frictionless circular track of radius  $R$ . Denote the positions of the particles by the angles  $\theta, \gamma$ . Define point  $O$  as the center of the circular track. Point  $B$  is the location of the particle of mass  $m$ .



- (a) Set up the problem. Define all appropriate unit vectors and locate the center of mass. Sketch the system FBD.

- (b) Derive the EOM for the system.

$$\left[ \begin{array}{l} \text{Ans: } R\ddot{\theta} + \frac{2}{3}L(\dot{\gamma} + \dot{\theta})\cos\gamma - \frac{2}{3}L(\dot{\gamma} + \dot{\theta})^2\sin\gamma = 0 \\ 2L(\dot{\gamma} + \dot{\theta}) + 2R\ddot{\theta}\cos\gamma + 2R\dot{\theta}^2\sin\gamma = 0 \end{array} \right]$$

- (c) Total system energy is constant in this problem. Justify this statement.

Write an expression for total system energy.

- (d) Total angular momentum is conserved. For which angular momentum is that statement true? Which, if any, are constant and why:  ${}^i\bar{H}^O$ ;  ${}^i\bar{H}^{cm}$ ;  ${}^i\bar{H}^B$ ? Write an expression for each angular momentum quantities if not already available.

- (e) Let  $R = 3$  met and  $L = 5$  met. Let  $\theta(0) = 0^\circ$ ,  $\dot{\theta}(0) = 0$ ,  $\gamma(0) = 30^\circ$ ,  $\dot{\gamma} = -0.1$  rad/s.

Numerically integrate the EOM for at least 3 cycles of the angles. Plot the angles (in deg) as functions of time. Plot  $\gamma$  as a function of  $\theta$  (angles in deg).

- (f) If  $H = |{}^i\bar{H}^O|$ , plot  $\Delta E$  and  $\Delta H$ .

Describe what is happening to the system?

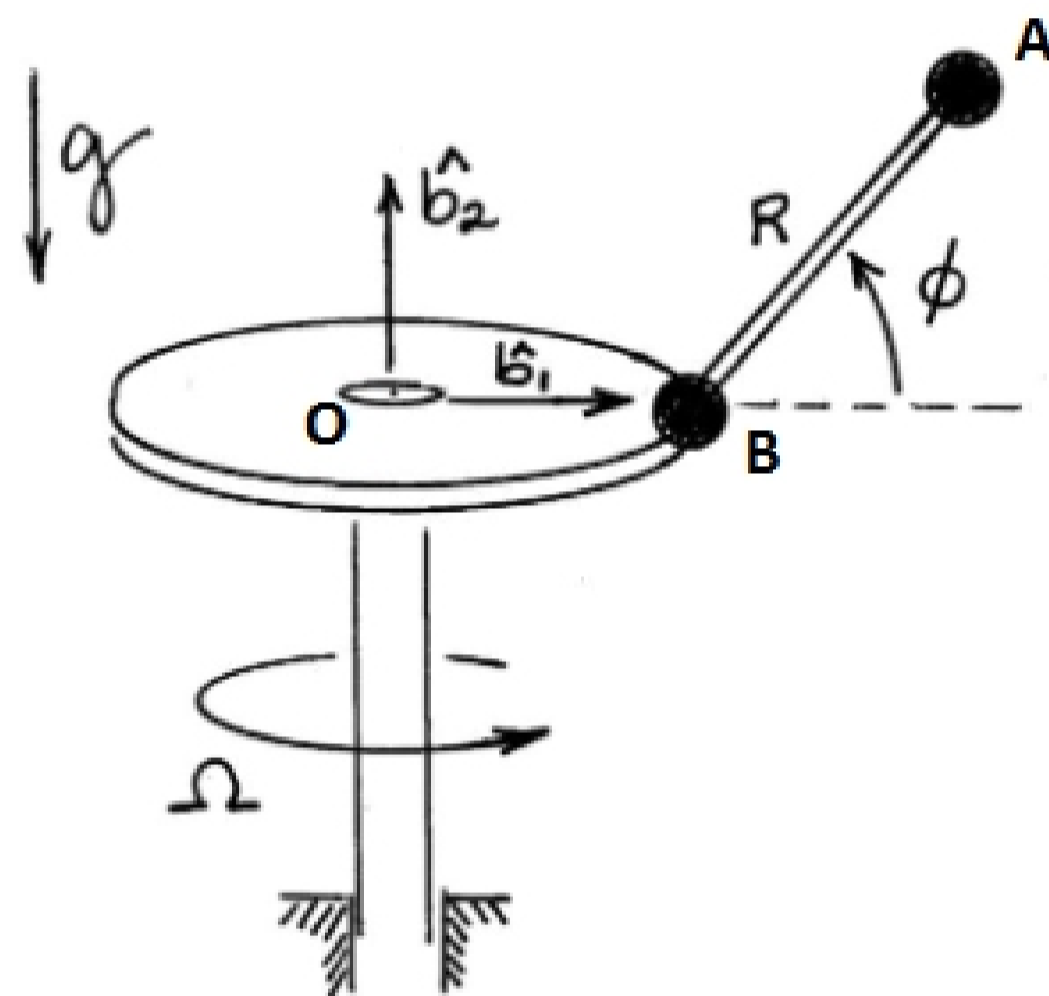
Max/min angles? Does  $\theta$  ever pass through  $360^\circ$ ?

**Problem 2:** The system below consists of two particles A and B of equal mass  $m$  connected by a rigid, “massless” inextensible rod of length  $R$ . One particle is attached along the edge of a “massless” thin disk of radius  $R$ . The rod is free to pivot in a vertical plane; the disk rotates about a vertical axis at the rate  $\Omega$  rad/s. The center of the disk  $O$  is on the axis of rotation. (Note that  $\Omega$  is constrained to be constant;  $\phi$  is variable.)

(a) Let  $\hat{i}$  be an inertial frame. For the points indicated, develop expressions for  ${}^i\bar{H}^O$ ,  ${}^i\bar{H}^{cm}$ , and  ${}^i\bar{H}^B$ . Are these equal?

(b) Derive expressions for  $\frac{d}{}^i dt \bar{H}^O$ ,  $\frac{d}{}^i dt \bar{H}^{cm}$ ,  $\frac{d}{}^i dt \bar{H}^B$ ,  $\mathcal{M} \bar{\rho}^{B \rightarrow cm} \times {}^i \bar{A}^B$ .

(c) Write expressions for  $T_{TRANS}$ ,  $T_{ROT+STR}$ .



**Problem 3:** The system below consists of two particles (masses  $m$  and  $2m$ ) fixed to the ends of a rigid, massless rod that is bent in the shape of a right angle. The rod is free to rotate in a vertical plane about the pivot point. A spring and dashpot are attached as indicated. When the rod is oriented as shown,  $\phi = 0^\circ$  and the system is in static equilibrium.

(a) Assume small oscillations about static equilibrium and derive the system EOM.

$$\left\{ \text{Ans: } \ddot{\phi} + \frac{4}{27} \frac{c}{m} \dot{\phi} + \left( \frac{k}{12m} + \frac{g \cos 45^\circ}{L} \right) \phi = 0 \right\}$$

(b) The system parameters have the following values:  $m = 2 \text{ kg}$ ;  $L = 5 \text{ meters}$ ;  $k = 48 \text{ N/m}$ ;  $c = 12 \text{ N-s/m}$ . Determine (if they exist) the values of the characteristic roots, natural frequency, damped frequency, damping ratio, period, static deformation of the spring. Is the spring in tension or compression at static equilibrium?

(c) At the initial time,  $\phi = 12^\circ$ ,  $\dot{\phi} = -0.05$ . Solve for  $\phi(t)$ .

(d) Plot the response of the system for 3 cycles; also add the other two relevant curves to your plot.

Small oscillations was assumed in the problem. Does the plot of the response support this assumption for these initial conditions?

