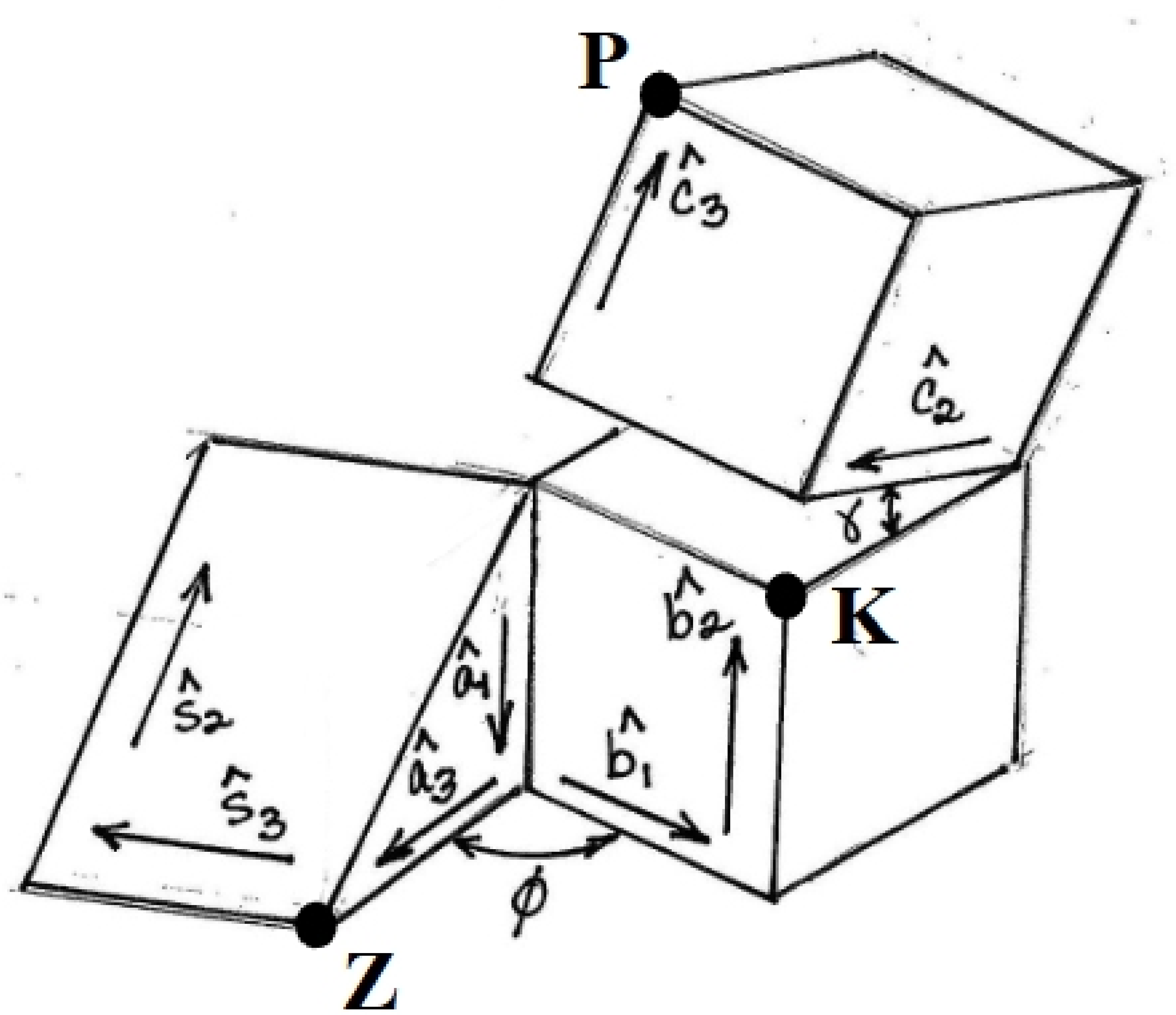
AAE 340 – Dynamics and Vibrations
Problem Set 2
Due: 9/4/13

Problem 1: Recall the cubes B and S and the wedge from PS1, with side length L. The same sets of unit vectors are fixed in each body.

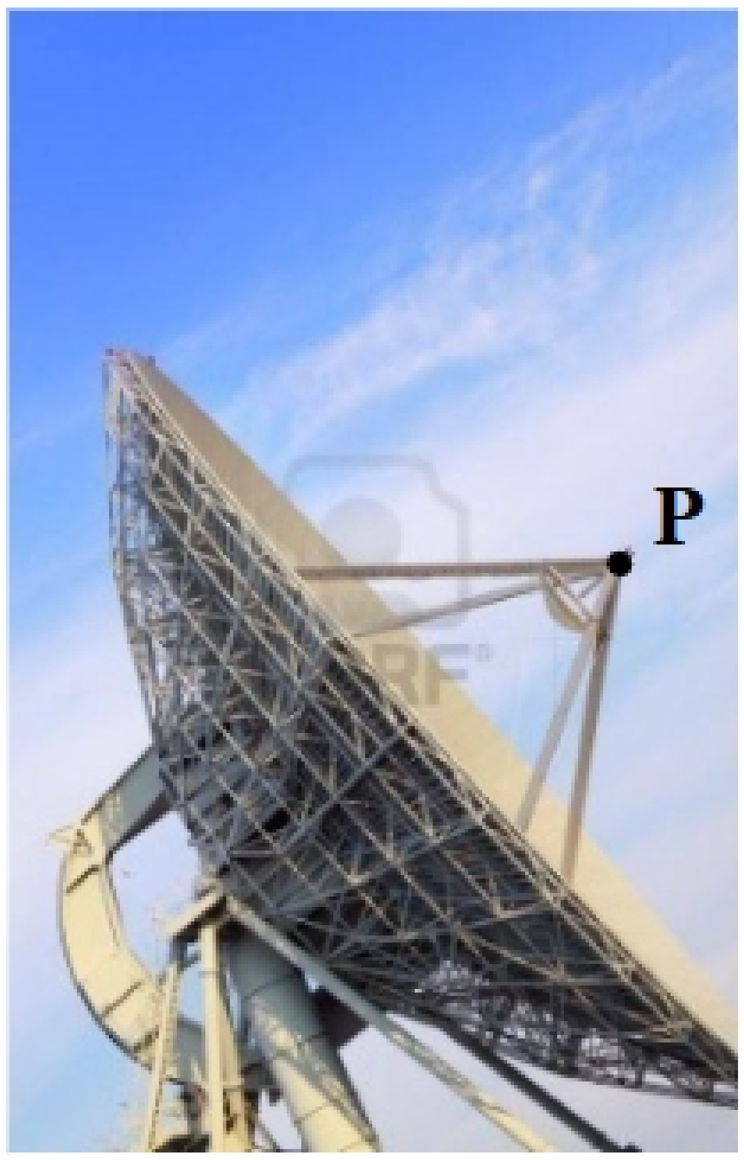
(a) Derive an expression for ${}^{a}\bar{v}^{KP}$ and express the result in terms of $\hat{b}$. (Use the BKE!)
Is this velocity generic? Why or why not?
Develop an expression for the acceleration ${}^{a}\bar{A}^{KP}$. Express it in terms of $\hat{b}$; $\hat{c}$.

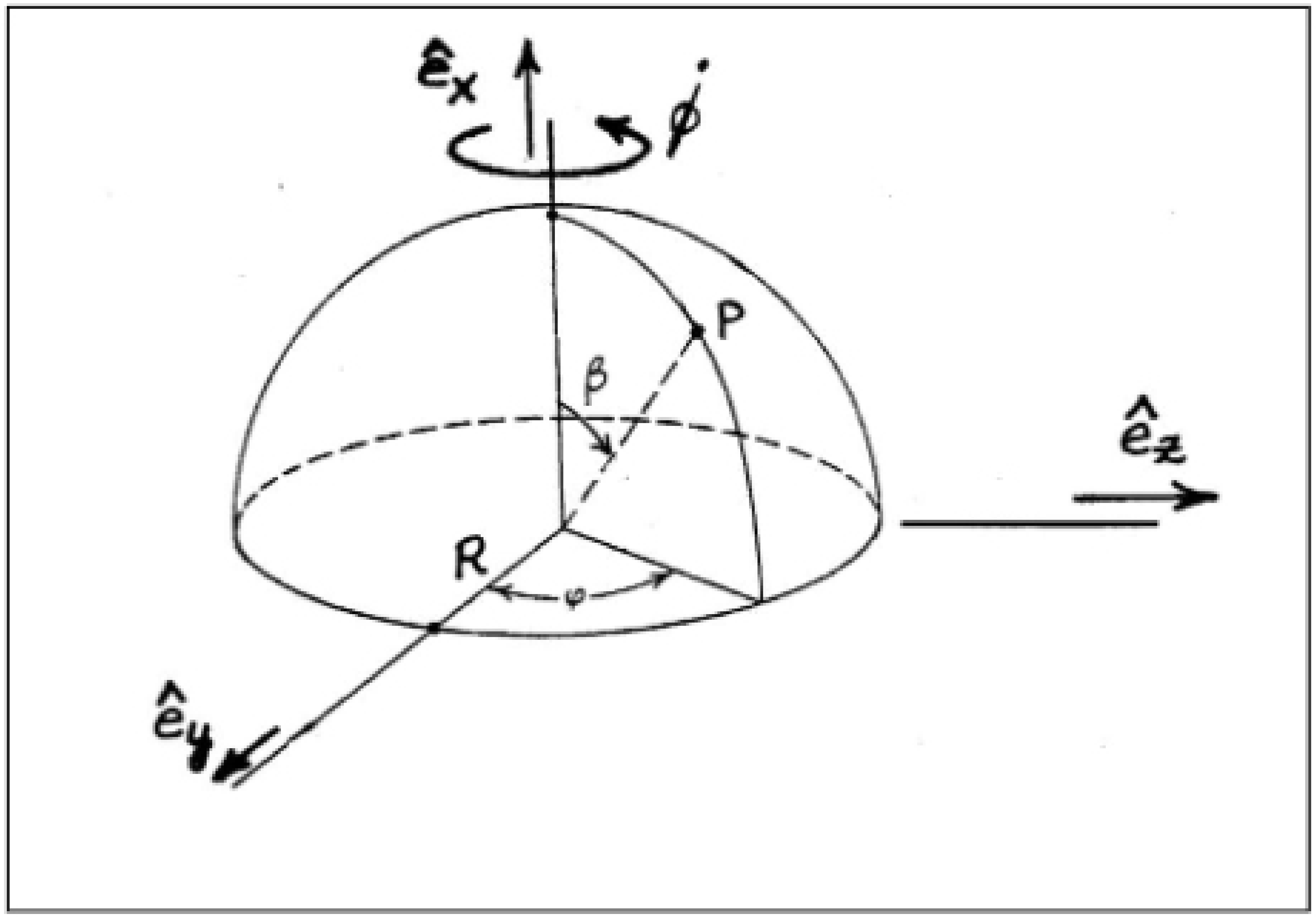
(b) Derive ${}^{s}\bar{v}^{K}$; also derive ${}^{c}\bar{v}^{K}$, Express both velocities in $\hat{c}$.
Does ${}^{s}\bar{v}^{K} = {}^{c}\bar{v}^{K}$? Why or why not?

(c) Derive ${}^{a}\bar{A}^{P}$, $\frac{{}^{b}d^2\bar{r}^{KP}}{dt^2}$, $\frac{{}^{a}d^2\bar{r}^{KP}}{dt^2}$. Express all results in terms of $\hat{b}$.
Are all the results equal? Different? If they are not the same, explain why not.



Problem 2: In a satellite tracking antenna system, assume that P is the focal point. The motion of P can be modeled as a point moving on a hemispherical surface such that the plane containing P rotates about its vertical axis at the angular rate $|\bar{\omega}| = \dot{\phi}$ as indicated. Assume that the radius $R = 5$ meter.





(a) Define the sets of unit vectors and variables to be consistent with both a cylindrical set of unit vectors as well as a set of spherical coordinates to locate the focal point at any time. For reference, use the inertial set of unit vectors $\hat{e}$ that are already defined.

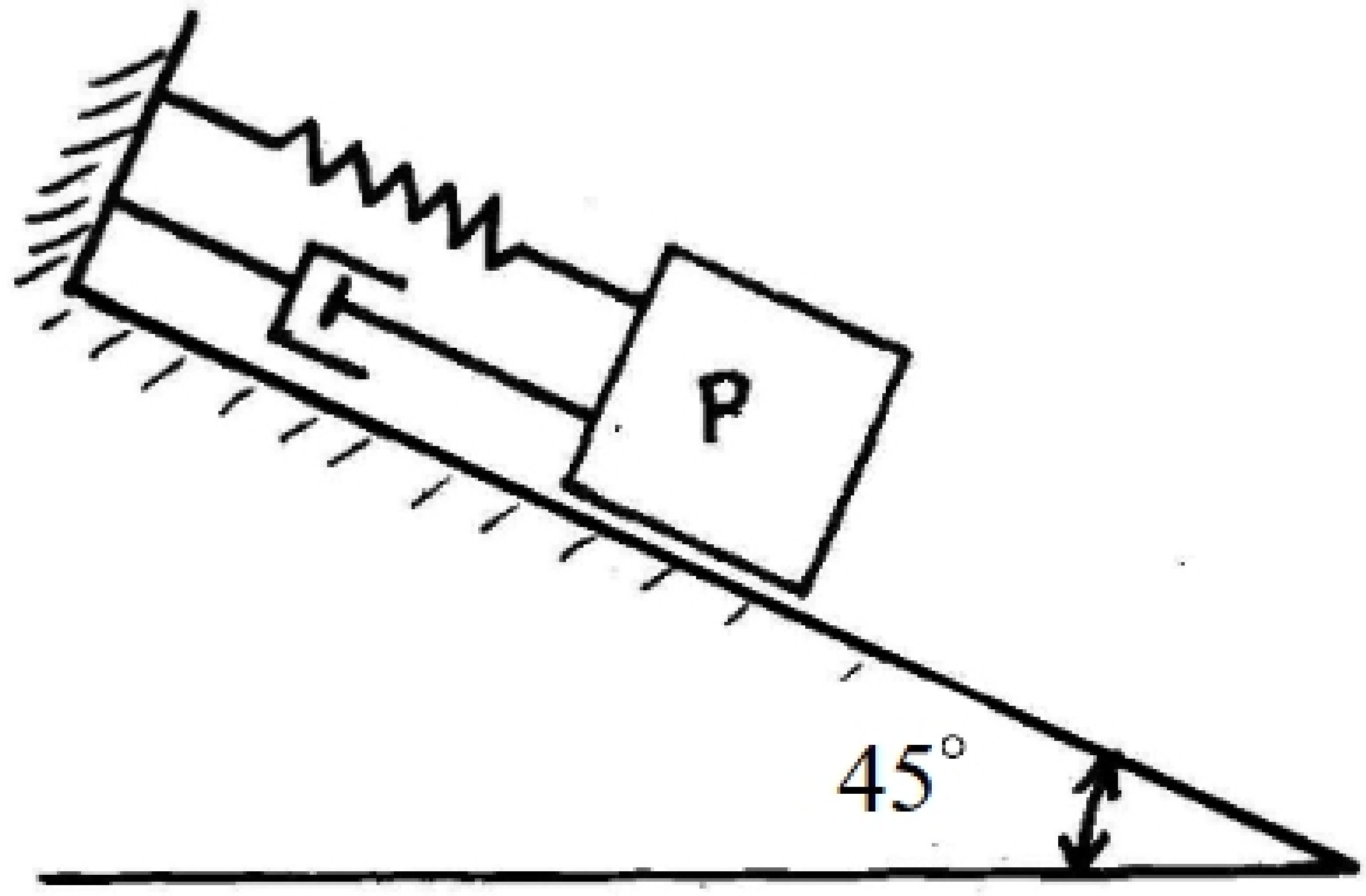
(b) Derive the general expressions for the generic acceleration of P relative to an inertial observer, i.e., ${}^{e}\bar{A}^{p}$.
Accomplish this derivation two ways:
(i) use a cylindrical set of unit vectors $\hat{u}$ as the working frame.
(ii) use a spherical set of unit vectors $\hat{a}$ as the working frame.
Express both results in terms of $\hat{a}$: demonstrate that the result is the same for both (i) , (ii).

(c) Evaluate this acceleration at the instant when

$$\dot{\phi} = 0 \text{ rpm}$$
$$\ddot{\phi} = -.002 \text{rad}/\text{s}^2$$
$$\phi = -10^\circ$$
$$\beta = 30^\circ$$
$$\dot{\beta} = .025 \text{deg}/\text{s}$$
$$\ddot{\beta} = -0.005 \text{deg}/s^2$$

(d) Write the general expression for ${}^{e}\bar{\omega}^{a}$ in terms of spherical, cylindrical, and inertial unit vectors. Evaluate the components in each case; demonstrate that the magnitude is equal.

Problem 3: In the figure below is a particle (mass 1.5 kg) that can move on a smooth inclined surface. A spring and dashpot are attached. Define y as the distance (in meters) of the particle along the surface relative to the **unstretched length of the spring.** Define u as the distance of the particle along the inclined surface from the **position of static equilibrium**. Assume that the dashpot constant $c = 3$ kg/s and the spring coefficient $k = 6$ kg/s^2.



(a) Set up the problem by defining all appropriate quantities in the diagram (unit vectors, variables of interest, reference definition).

(b) Develop the FBD and the force models.

(c) Derive the equation of motion in terms of both y and u. Why does gravity (g) appear in one equation and not the other?

$$\left[\begin{array}{c} Ans: \ \ddot{y} + 2\dot{y} + 4y = \dfrac{g}{\sqrt{2}} \\ \ddot{u} + 2\dot{u} + 4u = 0 \end{array} \right]$$