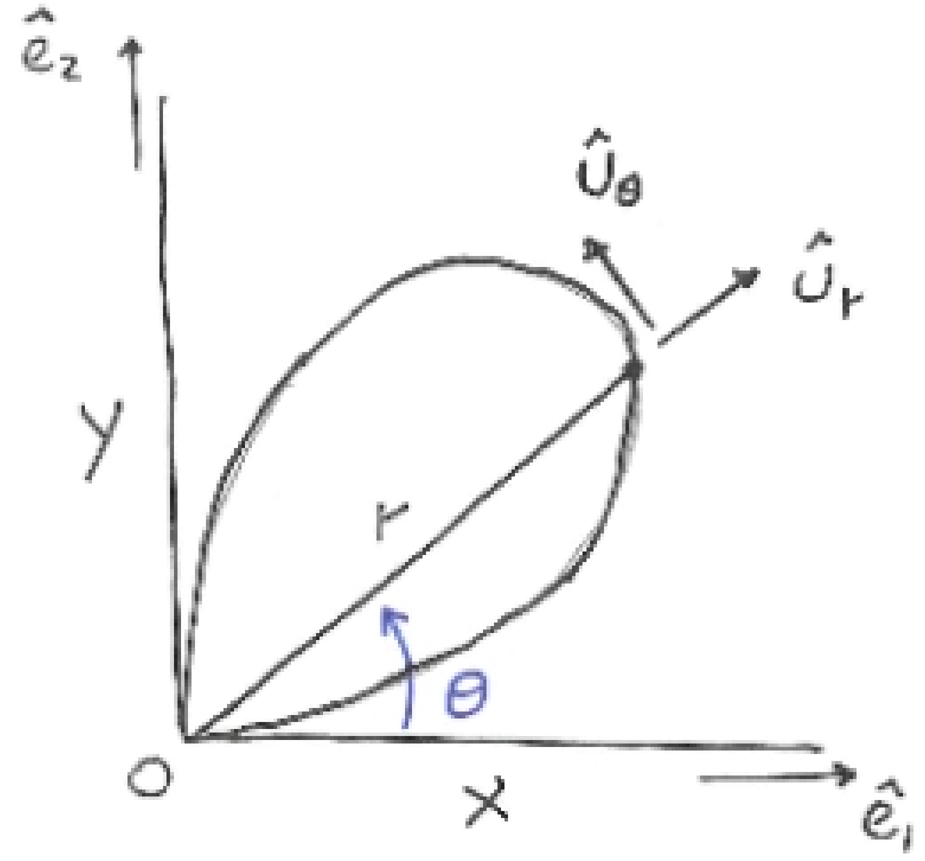
AAE 340 - PS 2 S13 SOLUTION DAVIDE GUZZETTI

PROBLEM 1

Given



- unit inertial vectors $\hat{e}$
- unit cylindrical vectors $\hat{u}$
- $r = C\sin(2\theta)$

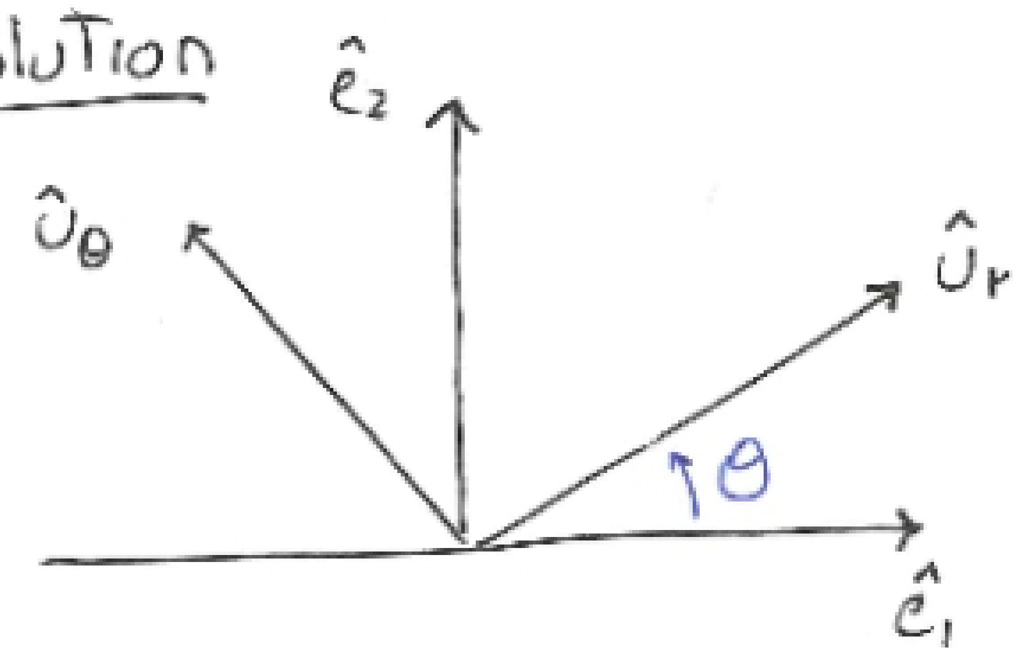
Find (a) $\hat{e}$ in terms of $\hat{u}$
$\hat{u}$ in terms of $\hat{e}$
$\bar{r}^{OP}$ in terms of $\hat{e}$ and $\hat{u}$

(b) ${}^{e}\bar{V}^{OP}$ in terms of $\hat{e}$ and $\hat{u}$ ($\hat{e}$ working frame)

(c) ${}^{e}\bar{V}^{OP}$ in terms of $\hat{e}$ and $\hat{u}$ ($\hat{u}$ working frame)
${}^{e}\bar{\omega}^{u}$
Compare the velocities obtained in pt (b) and (c) and draw appropriate conclusions.

(d) $|{}^{e}\bar{V}^{OP}|$, does the working frame change the velocity magnitude?

Solution



(a)

- $\hat{u}_r = c_\theta\,\hat{e}_1 + s_\theta\,\hat{e}_2$
$\hat{u}_\theta = -s_\theta\,\hat{e}_1 + c_\theta\,\hat{e}_2$

- $\hat{e}_1 = c_\theta\,\hat{u}_r - s_\theta\,\hat{u}_\theta$
$\hat{e}_2 = s_\theta\,\hat{u}_r + c_\theta\,\hat{u}_\theta$

$\bar{r}^{OP} = r\,\hat{u}_r = r c_\theta\,\hat{e}_1 + r s_\theta\,\hat{e}_2$ *

(b) Using $\hat{e}$ as working Frame

$$\bar{r}^{OP} = r c_\theta \hat{e}_1 + r s_\theta \hat{e}_2$$

$$^{e}\bar{v}^{OP} = \frac{^{e}d\bar{r}^{OP}}{dt} = \dot{r} c_\theta \hat{e}_1 + \dot{r} s_\theta \hat{e}_2 + r\dot{\theta}(-s_\theta \hat{e}_1 + c_\theta \hat{e}_2)$$

$$= \underline{(\dot{r} c_\theta - r\dot{\theta} s_\theta)\hat{e}_1 + (\dot{r} s_\theta + r\dot{\theta} c_\theta)\hat{e}_2} \quad *$$

Substituting $\hat{e}$ in terms of $\hat{u}$

$$^{e}\bar{v}^{OP} = (\dot{r} c_\theta - r\dot{\theta} s_\theta)\cdot(c_\theta \hat{u}_r - s_\theta \hat{u}_\theta) + (\dot{r} s_\theta + r\dot{\theta} c_\theta)(s_\theta \hat{u}_r + c_\theta \hat{u}_\theta)$$

$$= (\dot{r} c_\theta^2 - \cancel{r\dot{\theta} s_\theta c_\theta} + \dot{r} s_\theta^2 + \cancel{r\dot{\theta} s_\theta c_\theta})\hat{u}_r + (-\cancel{\dot{r} s_\theta c_\theta} + r\dot{\theta} s_\theta^2 + \cancel{\dot{r} s_\theta c_\theta} + r\dot{\theta} c_\theta^2)\cdot \hat{u}_\theta$$

$$= \dot{r}\underbrace{(c_\theta^2 + s_\theta^2)}_{1\ !!}\hat{u}_r + r\dot{\theta}\hat{u}_\theta = \underline{\dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta} \quad *$$

(c) Using $\hat{u}$ as working Frame

$$\bar{r}^{OP} = r\hat{u}_r$$

$$^{e}\bar{v}^{OP} = \frac{^{e}d\bar{r}^{OP}}{dt} = \frac{^{u}d\bar{r}^{OP}}{dt} + {}^{e}\bar{\omega}^{u} \times \bar{r}^{OP}$$

we need to use the BKE because the working Frame (that serves as basis For the position vector) is different From the observing Frame to take the position derivative.

$$\frac{^{u}d\bar{r}^{OP}}{dt} = \dot{r}\hat{u}_r$$

$$^{e}\bar{\omega}^{u} = +\dot{\theta}\hat{u}_z \qquad (\text{where } \hat{u}_z = \hat{u}_r \times \hat{u}_\theta)$$

$$^{e}\bar{\omega}^{u} \times \bar{r}^{OP} = r\dot{\theta}(\hat{u}_z \times \hat{u}_r) = r\dot{\theta}\hat{u}_\theta$$

Thus, substituting all Terms in the BKE

$$ {}^{e}\bar{V}^{OP} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta $$ *

Finally, substituting $\hat{u}$ in Terms of $\hat{e}$

$$ {}^{e}\bar{V}^{OP} = \dot{r}\,c_\theta\,\hat{e}_1 + \dot{r}\,s_\theta\,\hat{e}_2 + r\dot{\theta}(-s_\theta\,\hat{e}_1) + r\dot{\theta}\,c_\theta\,\hat{e}_2 $$

$$ = (\dot{r}\,c_\theta - r\dot{\theta}\,s_\theta)\,\hat{e}_1 + (\dot{r}\,s_\theta + r\dot{\theta}\,c_\theta)\,\hat{e}_2 $$ *

The point O is Fixed in the $\hat{e}$ Frame → ${}^{e}\bar{V}^{OP}$ is a generic velocity → generic velocity does not depend on the working frame → in (b) and (c) we end up w/ the same expression for (b), as it should be *

(d) Given

$r = C\sin(2\theta)$

$C = 4$ ft

$\dot{\theta} = 10$ deg/s $= 0.1745 \frac{rad}{s}$ *

we derive

$$ \theta(t=6sec) = \dot{\theta}t\Big|_{t=6sec} = 60 \text{ deg} $$

$$ \dot{r} = 2C\dot{\theta}\cos(2\theta)\Big|_{t=6sec} = 2\cdot 4\,ft \cdot .1745\,\frac{rad}{s}\left(-\frac{1}{2}\right) = -.698\,\frac{ft}{s} $$

$$ r = C\sin(2\theta)\Big|_{t=6sec} = 4\,ft\cdot\frac{\sqrt{3}}{2} = \sqrt{3}\cdot 2\ ft $$