

Math 2270 Extra Credit Problems
Chapter 5
November 2008

Due date: See the internet due date for 7.6, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems Extra Credit. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem Xc5.1-10. (Angle)

For which values of k are the vectors $\mathbf{u} = \begin{pmatrix} 2k \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ perpendicular?

Problem Xc5.1-26. (Orthogonal Projection)

Find the orthogonal projection of $\mathbf{w}$ onto the subspace V , given

$$\mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad V = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

Problem Xc5.1-34. (Minimization)

Among all the vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathcal{R}^3$, find the one with unit length that minimizes the sum $x + 2y + 3z$.

Problem Xc5.2-14. (Gram-Schmidt Basis)

Given the basis below, labeled $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, find the Gram-Schmidt basis $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

$$\begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 15 \\ 2 \\ 13 \end{pmatrix}.$$

Problem Xc5.2-20. (QR-Factorization)

Find the factorization $M = QR$, given

$$M = \begin{pmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{pmatrix}.$$

Problem Xc5.2-34. (Kernel)

Find an orthonormal basis for the kernel of the matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{pmatrix}$.

Problem Xc5.2-38. (QR-Factorization)

Find the factorization $M = QR$, given $M = \begin{pmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

Problem Xc5.3-11. (Orthogonal Matrices)

Given A and B are orthogonal, then which of the following must be orthogonal?

(a) $2A$, (b) ABA , (c) $A^{-1}B^T$, (d) $A - AB$, (e) $AB + BA$, (f) $-BA$

Problem Xc5.3-20. (Symmetric Matrices)

Given A and B are symmetric matrices and A is invertible, then which of the following must also be symmetric?

(a) $A^T A$, (b) ABA , (c) $A^{-1}B$, (d) $A - B$, (e) $A - BA$, (f) $A - A^T$, (g) $A^T B^T BA$, (h) $B(A + A^T)B^T$

Problem Xc5.3-26. (Dot Product)

Let T be an orthogonal transformation from $\mathcal{R}^n$ to $\mathcal{R}^n$. Prove that $\mathbf{u} \cdot \mathbf{v} = T(\mathbf{u}) \cdot T(\mathbf{v})$ for all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathcal{R}^n$.

Problem Xc5.3-32a. (Orthogonal Matrices)

Assume A is $n \times m$ and $A^T A = I$. Is AA^T the identity matrix? Explain.

Problem Xc5.3-44. (Orthogonal Matrices)

Consider an $n \times m$ matrix A . Find in terms of n and m the value of the sum $\text{rank}(\text{dim}(A)) + \text{rank}(\ker(A^T))$.

Problem Xc5.3-50. (QR-Factorization)

- (a) Find all square matrices A that are both orthogonal and upper triangular with positive diagonal entries.
- (b) Show that the QR -factorization is unique for an invertible square matrix A . Hint: see Exercise 50b in **Bretscher 3E**, section 5.3.

Problem Xc5.4-5. (Basis of $V^\perp$)

Find a basis for $V^\perp$, where $V = \ker(A)$ and

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Problem Xc5.4-16. (Rank)

Prove or disprove: The equation $\text{rank}(A) = \text{rank}(A^T A)$ hold for all square matrices A .

Problem Xc5.4-22. (Least Squares)

Find the least squares solution $\mathbf{x}^*$ of the system $A\mathbf{x} = \mathbf{b}$, given

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \\ 4 & 5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 10 \\ 18 \\ 4 \end{pmatrix}.$$

Problem Xc5.4-26. (Least Squares)

Find the least squares solution $\mathbf{x}^*$ of the system $A\mathbf{x} = \mathbf{b}$, given

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

Problem Xc5.5-10. (Orthonormal Basis)

Find an orthonormal basis for $V^\perp$, where $V = \text{span}\{1 + t^2\}$, in the space W of all polynomials $a_0 + a_1 t + a_2 t^2$ with inner product $\langle f, g \rangle = \frac{1}{2} \int_{-1}^1 f(t)g(t)dt$.

Problem Xc5.5-24. (Orthonormal Basis)

Consider the linear space P of all polynomials with inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. Let f, g, h be three polynomials satisfying the relations

$$\begin{array}{lll} \langle f, f \rangle = 4 & \langle f, g \rangle = 0 & \langle f, h \rangle = 8 \\ \langle g, f \rangle = 0 & \langle g, g \rangle = 2 & \langle g, h \rangle = 4 \\ \langle h, f \rangle = 8 & \langle h, g \rangle = 4 & \langle h, h \rangle = 10 \end{array}$$

- (a) Find $\langle f, g + 2h \rangle$.
- (b) Find $\|g + h\|$.
- (c) Find c_1, c_2 satisfying $\text{proj}_{\text{span}\{f, g\}}(h) = c_1 f + c_2 g$.
- (d) Find an orthonormal basis for the span of f, g, h expressed as linear combinations of f, g and h .

End of extra credit problems chapter 5.