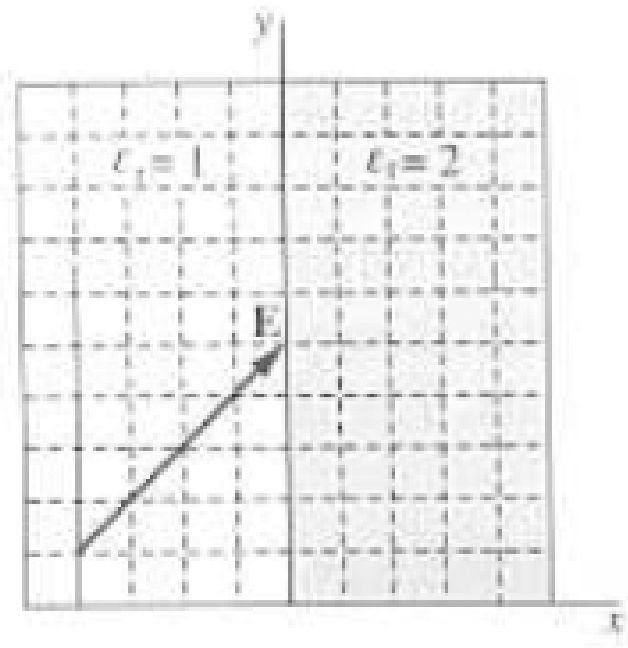


HW 7. 4.1.1
 4.1.2
 4.1.3
 4.1.4
 4.1.5
 4.1.6
 4.1.7
 Analytical
 Numerical
 $h = 1/3$
 $h = 1/4$
 $h = 1/5$

Written using MATLAB are available at the Web <http://www.comsol.com/lonngren2e.htm>.
 In this chapter, all of the methods assume linearity, which is a simplification of the *Maxwell equations*. However, you will frequently encounter *nonlinear materials*, which will lead to significant alterations in the solution. Numerical questions concerning other languages, convergence requirements, numerical stability, etc. are left to later courses. As problems arise, hopefully you will find the solutions.

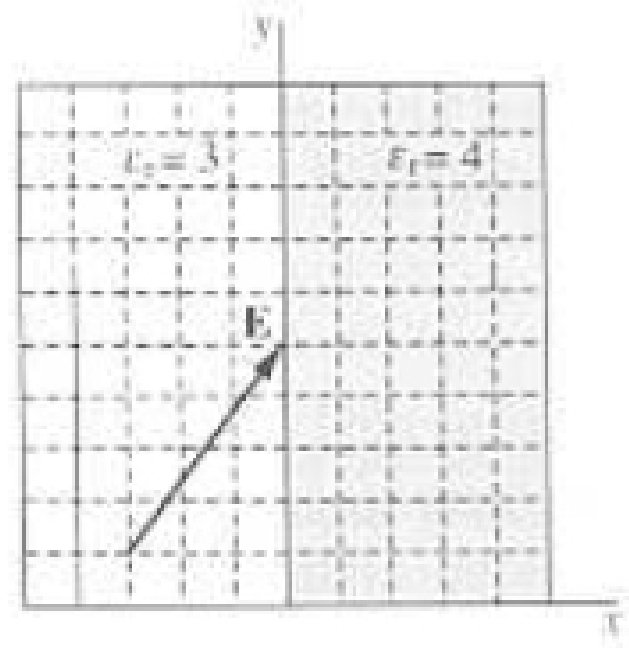
4.10 Problems

4.1.1. Find the electric field in the region $x > 0$ if $\mathbf{E} = 2\mathbf{u}_x + 2\mathbf{u}_y$ V/m in the region $x < 0$. There is no surface charge density.



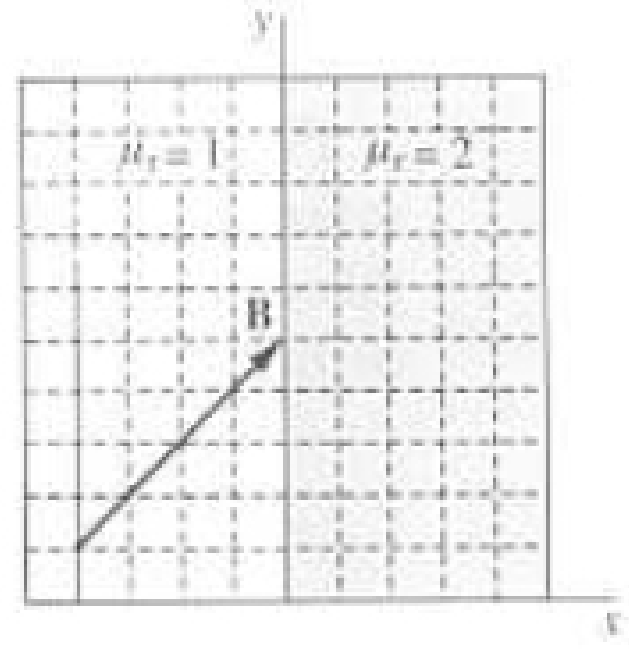
4.1.2. Repeat Problem 4.1.1 with a surface charge density $\rho_s > 1$ C/m².

4.1.3. Find the electric field in the region $x > 0$. There is no surface charge density. The magnitude of the electric field is 5 V/m.



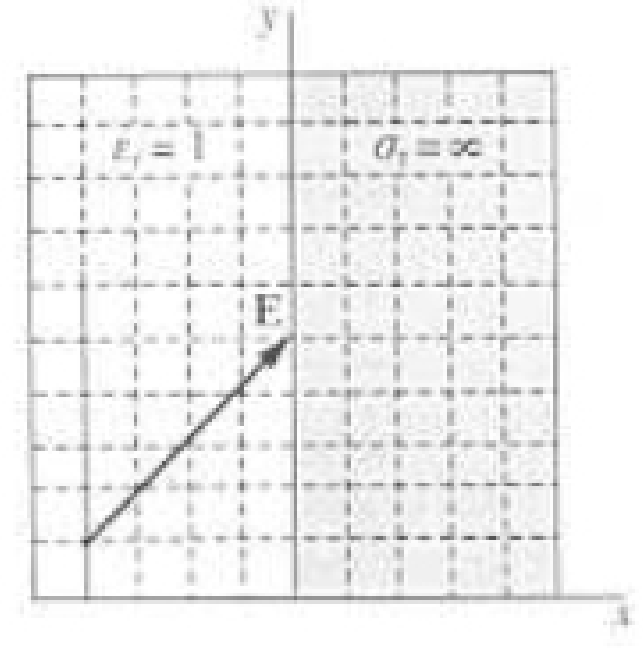
4.1.4. Repeat Problem 4.1.3 with a surface charge density $\rho_s > 0.5$ C/m².

4.1.5. Find the magnetic flux density in the region $x > 0$ if $\mathbf{B} > 4\mathbf{u}_x + 4\mathbf{u}_y$ T in the region $x < 0$. The surface current equals zero.

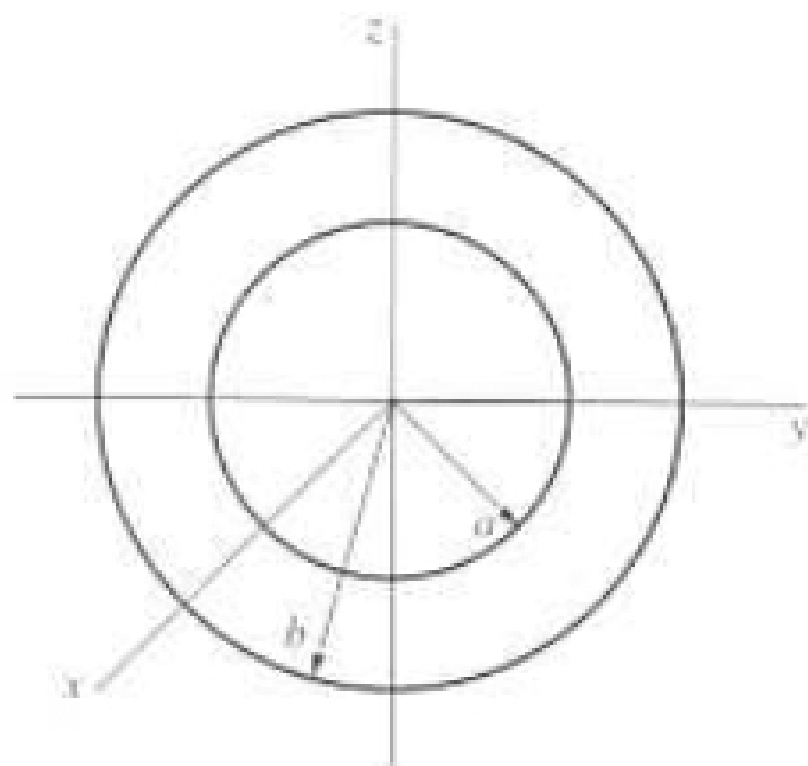


4.1.6. Repeat Problem 4.1.5 if the surface current density $\mathbf{J}_L = (9\mathbf{u}_x + 9\mathbf{u}_y)$ A/m.

4.1.7. Can the indicated electric field exist? If not, suggest how an additional electric field will permit the existence of this field.

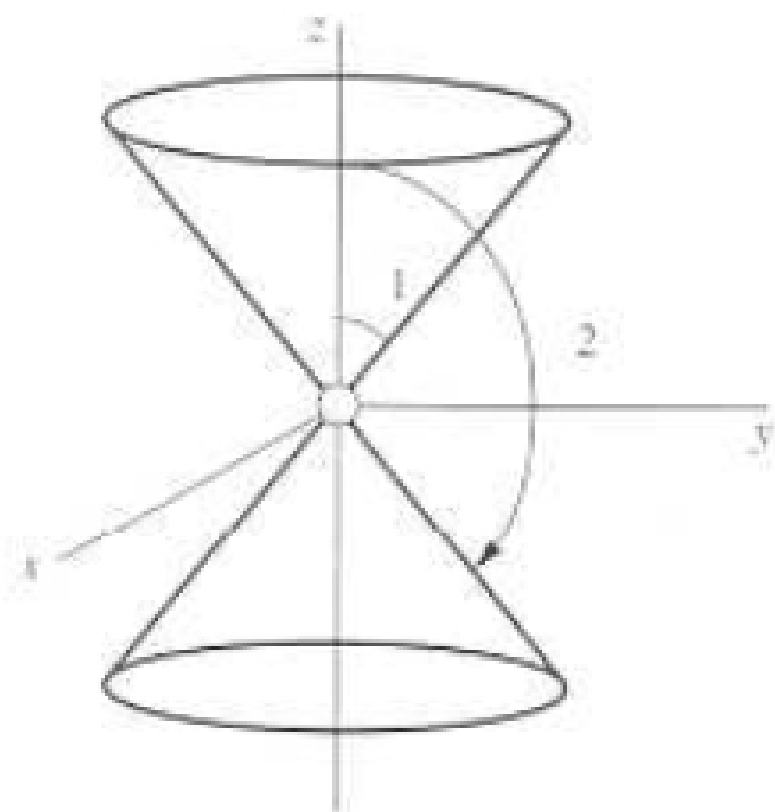


4.3.1. Find the potential distribution $V(r)$ by solving Laplace's equation analytically for the region between two concentric hollow spheres (spherical capacitor). Apply a spherical coordinate system with the following boundary conditions: $V = 0$ at $r = a$ and $V = V_0$ at $r = b$. Simplify the calculation with symmetry arguments.



4.3.2. Find the capacitance C of the spherical capacitor in Problem 4.3.1.

4.3.3. Find the potential distribution $V(\theta)$ by solving Laplace's equation analytically for the region between two hollow coaxial cones. A potential $V = V_1$ is assumed at $\theta = \theta_1$ and $V = 0$ at $\theta = \theta_2 = \pi - \theta_1$. The vertices of the cones are insulated at $\rho = 0$. Simplify the calculation with symmetry arguments.



4.3.4. Find the potential $V(x)$ in the region $0 < x < 1$ satisfying the boundary conditions $V(0) = 3$ and $V(1) = 0$.

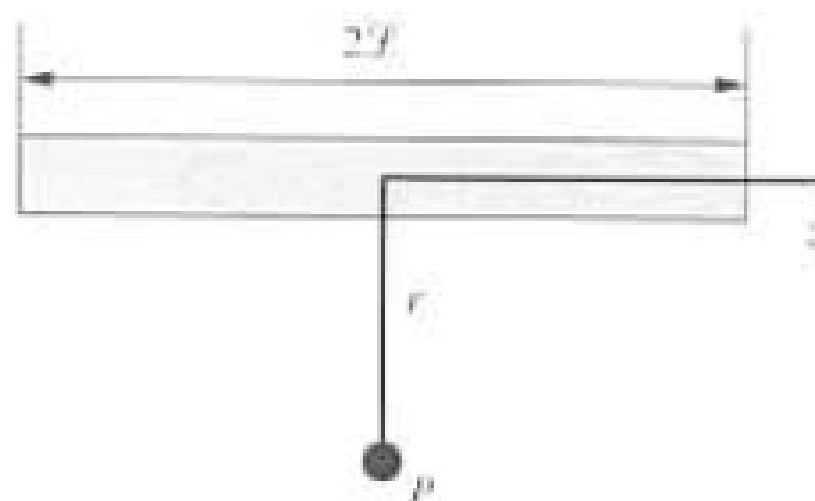
4.3.5. Find the potential $V(x)$ in the region $0 < x < 2$ if the electric field is normal with a constant value $E_x = 4$.

4.3.6. Find the potential $V(x)$ in the region $0 < x < 1$. Assume that a charge distributed uniformly there has a density $\rho_v = -4\epsilon$. The potential satisfies the boundary conditions: $V(0) = 3$ and $V(1) = 0$.

4.3.7. Find the normal electric field $E_x(x)$ in Problem 4.3.6.

4.3.8. Find the capacitance C_0 of unit length of the cylindrical capacitor—two long concentric cylinders with radii a and b ($b > a$). The boundary conditions for the potential are $V(\rho = a) = V_0$ and $V(\rho = b) = 0$.

4.3.9. Compute using an analytical integration, the potential $V(r)$ at the point P that is a distance $r = 1$ m from the midpoint of a narrow finite strip that has a length $2\mathcal{L} = 2$ m. A charge of 1C is distributed uniformly on the strip.



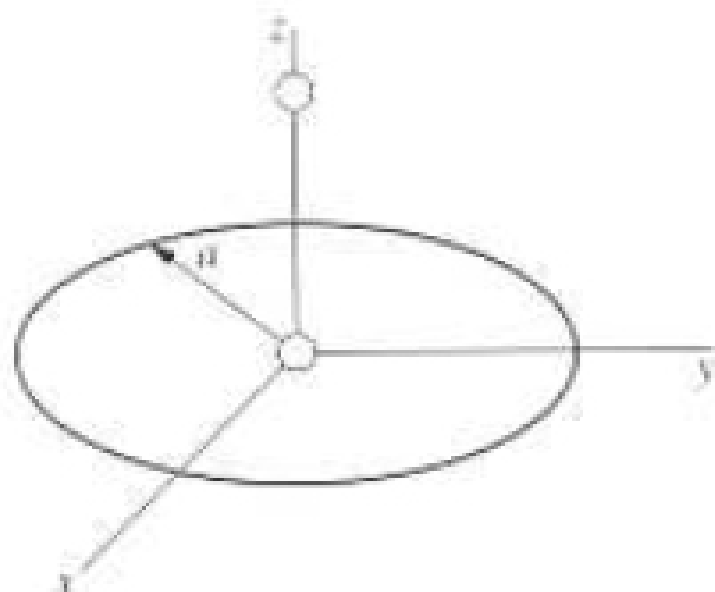
4.3.10. Repeat Problem 4.3.9 with a nonuniform charge distribution $\rho_v(z) = \rho_{v0}[1 - |z|/\mathcal{L}]$ where $z = 0$ is at the midpoint of the strip.

4.3.11. Repeat Problem 4.3.9 but compute the electric field $E_r(r)$ instead of the potential.

4.3.12. Compare the numerical and analytical integrations that lead to the potential $V(z)$ along the z axis from the charged circular loop with a diameter $2a$. Charge Q is distributed uniformly upon the loop.

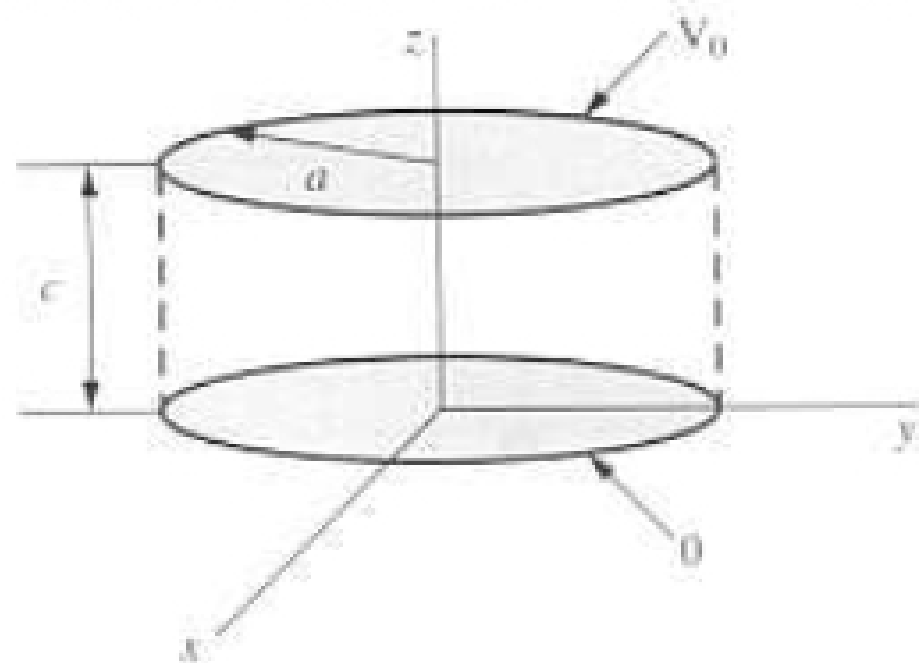
4.3.13. Compare the results obtained from a numerical and an analytical integration for the electric field $E_z(z)$ in the previous problem. Find the solution in the limiting case $a \rightarrow 0$.

4.3.14. Compare the results of a numerical and analytical analysis for the potential $V(z)$ along the z axis from the charged circular plate with a diameter $2a$. Charge Q is distributed uniformly upon the plate.

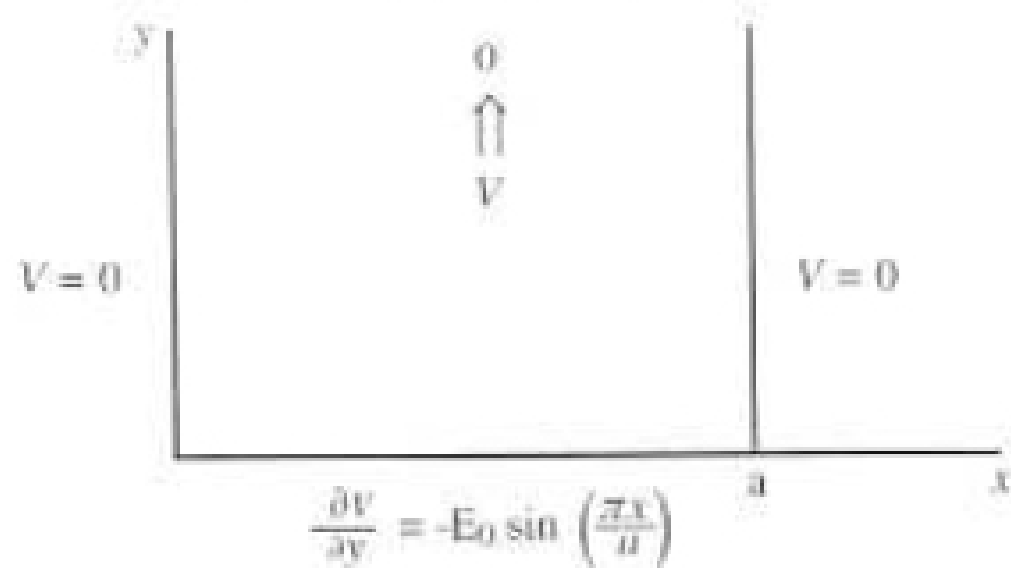


4.3.15. Compare the results of a numerical and analytical analysis for the potential $E_z(z)$ in the previous problem. Find the solution in the limiting case $a \rightarrow \infty$.

4.3.16. Compare the results of a numerical and analytical analysis for the potential $V(z)$ between two parallel discs ($0 < z < c$) that have a large enough radius a such that the electric field is constrained to be entirely between them. This implies that the fringing fields are neglected. The boundary conditions are $V(0) = 0$ and $V(c) = V_0$. Find the normal electric field E_z , the flux density D_z , and the surface charge density ρ_s on both plates.



4.5.1. For the indicated boundary conditions that are specified in the figure, find the potential distribution $V(x, y)$ within the enclosed region by solving Laplace's equation and expanding one boundary condition in a Fourier series expansion.



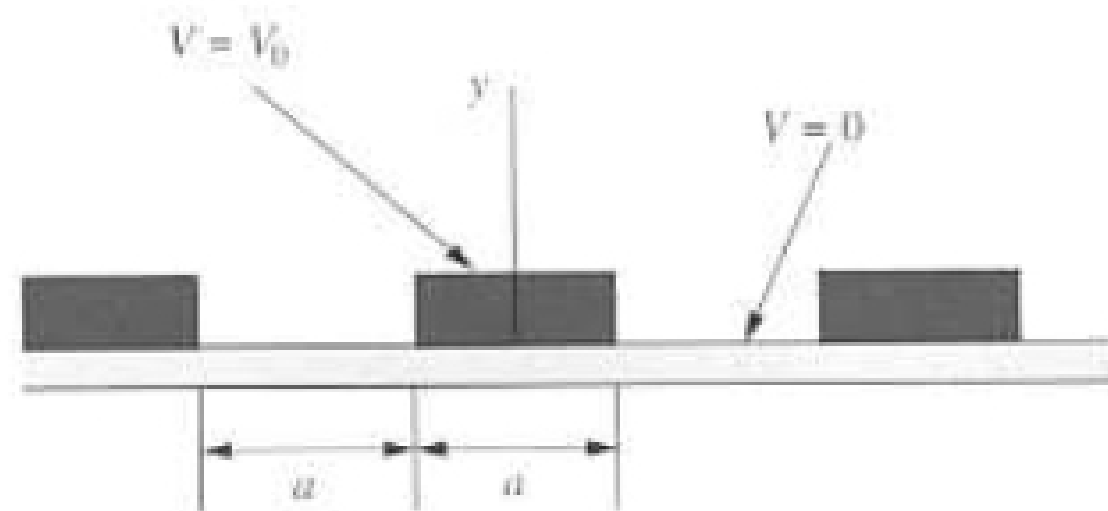
4.5.2. Find the potential within the channel given in Problem 4.5.1 using the boundary conditions at $x = 0$, $x = a$, and $y = \infty$ as stated there. The boundary condition at $y = 0$ is given as $E = E_0 u_y$ where E_0 is a constant.

4.5.3. Find the potential within the channel given in Problem 4.5.1 using the boundary conditions at $x = 0$, $x = a$, and $y = \infty$ as stated there. The boundary condition at $y = 0$ is: $V_0 = +V_0$ for $0 < x < (a/2)$ and $V = -V_0$ for $(a/2) < x < a$.

4.5.4. Find the components of the electric field (E_x, E_y) in Problem 4.5.3.

4.5.5. Find the expression for a potential $V = V_0(x/a)$ that describes the potential variation in the region $0 > x > a$.

4.5.6. Find the potential distribution $V(x, y)$ in the region $y > 0$. Because of the periodicity of the boundary condition, expand boundary in a Fourier series. This problem models a VLSI circuit where conductors are implanted on an insulating material. The thickness of the metal strip can be neglected.



4.5.7. For the indicated boundary conditions that are specified in the figure, find the potential distribution $V(x, y)$ within the enclosed region by solving Laplace's equation. Plot the potential distribution ($a = 1$ m, $V_0 = 25$ V).

