Lecture #2 EGR 261 – Signals & Systems

Read: Ch. 12, Sect. 1-9 in *Electric Circuits, 9th Edition* by Nilsson

Handout: Laplace Transform Properties and Common Laplace Transform Pairs

Laplace Transform Properties

Laplace transforms of complicated functions may be found by using known transforms of simple functions and then by applying properties in order to see the effect on the Laplace transform due to some modification to the time function.

Table of Laplace Transform Properties

1	Linearity	$\mathcal{L}\{af(t)\} = aF(s)$
2	Superposition	$\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$
3	Modulation	$\mathcal{L}\{e^{-at}f(t)\} = F(s+a)$
4	Time-Shifting	$\mathcal{L}\{f(t-\tau)u(t-\tau)\} = e^{-s\tau}F(s)$
5	Scaling	$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
6	Real Differentiation	$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0)$
7	Real Integration	$\mathcal{L}\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$
8	Complex Differentiation	$\mathcal{L}\{tf(t)\} = -\frac{d}{ds}F(s)$
9	Complex Integration	$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds$
10	Convolution	$\mathcal{L}\{f(t) * g(t)\} = F(s) \cdot G(s)$

Laplace Transform Properties: (continued)

6. **Real (Time) Differentiation:** $$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0)$$

Example: Find **L** {f '(t)}

Example: Find **L** {f ''(t)}

Example: Find **L** {f '''(t)}

Example: Find **L** {f [n](t)}

Example: Find the Laplace transform of the relationship: $i(t) = C\frac{dv}{dt}$

7. **Real (Time) Integration**: $$\mathcal{L}\left[\int_0^t f(t)dt\right] = \frac{1}{s}F(s)$$

Example: Find $\mathcal{L}\left[\int_0^t\int_0^t f(t)dtdt\right]$