
Major results: (wrt propositional logic)

- How to reason correctly.
- How to reason efficiently.
- How to determine if a statement is true or false.

Fuzzy logic deal with statements that are somewhat vague, such as: this paint is grey.

Probability deals with statements that are possibly true, such as whether I will win the lottery next week.

Propositional logic deals with concrete statements that are either true or false.

Predicate Logic (first order logic) allows statements to contain variables, such as if X is married to Y then Y is married to X.

We need formal definitions for:

- legal sentences or propositions (syntax)
- inference rules, for generated new sentence. (entailment)
- truth or validity. (semantics)

Example

Syntax: $(x > y) \& (y > z) \rightarrow (x > z)$

Interpretation: $x =$ John's age

$y =$ Mary's age

$z =$ Tom's age

If John is older than Mary and Mary is older than Tom, then we can conclude John is older than Tom. (semantics)

Conclusion is not approximately true or probably true, but absolutely true.

1 Propositional Logic

Syntax:

logical constants True and False are sentences.

propositional symbols P, Q, R, etc are sentences

These will either be true (t) or false (f)

Balanced Parenthesis if S is a sentence, so is (S).

logical operators or connectives : $\vee, \wedge, \sim, \dots, \Rightarrow, \dots$

Actually NAND (Shaeffer stroke) is the only connective that is needed.

Sentences legal combinations of connectives and variables, i.e. propositional letters are sentences and if s1 and s2 are sentences so are $\sim s1, (s1), s1 \Rightarrow s2,$

Note: Instead of \Rightarrow one can use \vee and \wedge since $p \Rightarrow q \Leftrightarrow \sim p \vee q$

Semantics:

- Each propositional variable can be either True or False.
- The truth or falsity of a sentence is defined compositionally.
- Truth tables define the semantics of the logical operations.
- A sentence is true if any assignment of the propositional variables yields true.

This means that a formula with k variables will have 2^k interpretations.