

# CS530 Public Key Cryptography

Bill Cheng

*<http://merlot.usc.edu/cs530-s10>*



# Public Key Cryptography

- ➡ aka asymmetric cryptography
- ➡ Based on some NP-complete problem
  - = traveling salesman problem
    - $n$  cities, connected
    - find shortest tour, all cities must be visited
    - solution complexity is  $n!$
  - = unique factorization
    - factor an integer into product of prime numbers (unique solution)
  - = discrete logarithms
    - for any integers  $b, n, y$ : Find  $x$  such that  $b^x \bmod n = y$
    - modular arithmetic produces folding

## A Short Note on Primes

- ➡ Why are public keys (and private keys) so large?
  - = because key space is *sparse*
- ➡ What is the probability that some large number  $p$  is prime?
  - = about 1 in  $1/\ln(p)$ 
    - 2 digit numbers: 25 primes (1 in 4)
    - 10 digit numbers: 1 in 23 are primes
    - 100 digit numbers: 1 in 230 are primes
    - but... the more digits, the more primes!
  - = when  $p \approx 2^{512} (\approx 10^{150})$ , equals about 1 in 355
    - about 1 in  $355^2$  numbers  $\approx 2^{1024}$  is product of two primes (and therefore valid RSA modulo)